x^7-2x^5+x^4 +4x^3-x^2-4 x +3 の厳密解 with *Mathematica* 14

R_k の係数を使って Gal= F_{42}の方程式を解く

2025 年5月 by mixedmoss

注【計算量が多いので，ノートブックの評価には20秒ほどかかります】

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§0. [準備] 原始元, 分解方程式, ガロア群

SageMathを使って求めました．詳しくは `sagemath.shtml` をご参照ください．

Galois群は フロベニウス群$F_{42} = C_7\,C_6 = \langle \sigma, \tau \rangle$, 但し $\sigma = (1, 2, 3, 4, 5, 6, 7)$, $\tau = (1, 3, 2, 6, 4, 5)$, 分解列は $F_{42} \triangleright F_{21} \triangleright C_7 \triangleright \{id\}$,
但し$F_{21} = \langle \sigma, \tau^2 \rangle$. 或いは $F_{42} \triangleright D_7 \triangleright C_7 \triangleright \{id\}$, 但し $D_7 = \langle \sigma, \tau^3 \rangle$

In[∘]:=
```
ClearAll["`*"];
```

In[∘]:=
```
f[x_] = x^7 - 2 x^5 + x^4 + 4 x^3 - x^2 - 4 x + 3;
```

f(x) = 0 の解をx1~x7, 原始元の1つをv, vの最小多項式をV (x)とする.

In[∘]:=
```
V[x_] = x^42 - 28 * x^40 + 396 * x^38 - 3569 * x^36 + 21 260 * x^34 - 72 675 * x^32 + 10 544 * x^30 +
    1 246 661 * x^28 - 5 902 845 * x^26 + 7 569 068 * x^24 + 36 867 012 * x^22 - 157 414 881 * x^20 +
    22 205 417 * x^18 + 848 975 725 * x^16 - 503 974 224 * x^14 - 3 040 667 773 * x^12 + 404 109 589 * x^10 +
    6 330 002 991 * x^8 + 7 362 768 751 * x^6 + 3 706 246 726 * x^4 + 873 281 136 * x^2 + 127 353 499;
```

まずはvを使ってx1~x7を表し，その集合をsols={$x1,$x2...$x7} と置く．(出力は長いのでフォントは小さくしています)

In[∘]:=
```
sols = {$x1, $x2, $x3, $x4, $x5, $x6, $x7} =
```

$$\Big\{\frac{96\,632\,638\,432\,453\,628\,071\,050\,796\,796\,660\,223\,873\,313\,711\,316}{62\,609\,452\,658\,981\,321\,494\,654\,745\,708\,976\,627\,407\,962\,995\,501} + \frac{v}{2} + \frac{3\,637\,017\,704\,098\,182\,974\,627\,003\,435\,602\,279\,693\,868\,195\,305\,399\,v^2}{876\,532\,337\,225\,738\,500\,925\,166\,439\,925\,672\,783\,711\,481\,937\,014} + \frac{5\,011\,928\,958\,072\,860\,605\,104\,979\,990\,835\,090\,507\,297\,547\,003\,227\,v^4}{876\,532\,337\,225\,738\,500\,925\,166\,439\,925\,672\,783\,711\,481\,937\,014} +$$
$$\frac{1\,376\,615\,098\,014\,096\,136\,938\,460\,630\,402\,520\,413\,819\,930\,988\,119\,v^6}{292\,177\,445\,741\,912\,833\,641\,722\,146\,641\,890\,927\,903\,827\,312\,338} + \frac{240\,670\,522\,744\,669\,421\,572\,142\,229\,851\,572\,198\,924\,328\,653\,252\,v^8}{70\,339\,014\,715\,645\,682\,173\,007\,183\,450\,825\,593\,754\,625\,093\,711} - \frac{321\,815\,802\,559\,772\,789\,751\,607\,027\,647\,991\,936\,332\,144\,442\,343\,v^{10}}{125\,218\,905\,317\,962\,642\,989\,309\,491\,417\,953\,254\,815\,925\,991\,002} -$$
$$\frac{45\,566\,706\,873\,078\,835\,901\,734\,848\,929\,076\,638\,452\,747\,688\,724\,532\,v^{12}}{24\,688\,994\,165\,191\,634\,442\,725\,521\,391\,239\,783\,407\,873\,407\,892\,561} + \frac{24\,506\,886\,629\,878\,134\,026\,012\,747\,229\,041\,119\,152\,076\,506\,918\,488\,v^{14}}{24\,688\,994\,165\,191\,634\,442\,725\,521\,391\,239\,783\,407\,873\,407\,892\,561} + \frac{6\,665\,510\,819\,287\,368\,992\,592\,916\,284\,378\,586\,018\,031\,708\,720\,209\,v^{16}}{21\,161\,994\,998\,735\,686\,665\,193\,304\,049\,634\,100\,063\,891\,492\,479\,338} -$$
$$\frac{13\,782\,094\,473\,610\,302\,613\,153\,541\,442\,338\,082\,951\,083\,948\,881\,183\,v^{18}}{49\,377\,988\,330\,383\,268\,885\,451\,042\,782\,479\,566\,815\,746\,815\,785\,122} + \frac{22\,831\,089\,135\,151\,560\,353\,745\,064\,161\,417\,834\,647\,761\,128\,928\,273\,v^{20}}{641\,913\,848\,294\,982\,495\,510\,863\,556\,172\,234\,368\,604\,708\,605\,206\,586} +$$
$$\frac{1\,577\,717\,078\,663\,024\,528\,335\,174\,561\,058\,513\,334\,734\,442\,255\,241\,v^{22}}{71\,323\,760\,921\,664\,721\,723\,429\,284\,019\,137\,152\,067\,189\,845\,022\,954} - \frac{270\,680\,287\,253\,828\,846\,123\,927\,649\,788\,599\,297\,302\,178\,483\,889\,791\,v^{24}}{25\,034\,640\,083\,504\,317\,324\,923\,678\,690\,717\,140\,375\,583\,635\,603\,056\,854} +$$
$$\frac{22\,200\,136\,102\,960\,279\,235\,874\,120\,467\,635\,354\,497\,835\,586\,418\,433\,v^{26}}{12\,517\,320\,041\,752\,158\,662\,461\,839\,345\,358\,570\,187\,791\,817\,801\,528\,427} + \frac{1\,966\,735\,258\,194\,424\,748\,954\,054\,506\,479\,683\,118\,080\,722\,563\,827\,v^{28}}{12\,517\,320\,041\,752\,158\,662\,461\,839\,345\,358\,570\,187\,791\,817\,801\,528\,427} -$$
$$\frac{1\,238\,922\,646\,515\,362\,369\,789\,518\,418\,621\,953\,873\,382\,476\,290\,253\,v^{30}}{8\,344\,880\,027\,834\,772\,441\,641\,226\,230\,239\,046\,791\,861\,211\,867\,685\,618} + \frac{321\,536\,629\,370\,674\,449\,080\,807\,747\,790\,435\,100\,057\,582\,471\,955\,v^{32}}{8\,344\,880\,027\,834\,772\,441\,641\,226\,230\,239\,046\,791\,861\,211\,867\,685\,618} - \frac{153\,039\,971\,025\,005\,331\,660\,994\,874\,269\,660\,265\,473\,688\,335\,789\,v^{34}}{25\,034\,640\,083\,504\,317\,324\,923\,678\,690\,717\,140\,375\,583\,635\,603\,056\,854} +$$
$$\frac{2\,720\,579\,755\,772\,758\,196\,222\,763\,279\,229\,524\,636\,919\,622\,656\,v^{36}}{4\,172\,440\,013\,917\,386\,220\,820\,613\,115\,119\,523\,395\,930\,605\,933\,842\,809} - \frac{372\,108\,843\,164\,451\,880\,965\,088\,833\,131\,335\,627\,617\,449\,911\,v^{38}}{8\,344\,880\,027\,834\,772\,441\,641\,226\,230\,239\,046\,791\,861\,211\,867\,685\,618} + \frac{19\,299\,124\,153\,388\,721\,784\,979\,223\,864\,064\,323\,408\,935\,722\,v^{40}}{12\,517\,320\,041\,752\,158\,662\,461\,839\,345\,358\,570\,187\,791\,817\,801\,528\,427},$$
$$-\frac{88\,241\,033\,276\,128\,162\,996\,093\,511\,842\,393\,324\,406\,337\,916\,420}{62\,609\,452\,658\,981\,321\,494\,654\,745\,708\,976\,627\,407\,962\,995\,501} - \frac{827\,201\,936\,611\,387\,419\,702\,597\,540\,657\,036\,572\,443\,629\,638\,032\,v^2}{62\,609\,452\,658\,981\,321\,494\,654\,745\,708\,976\,627\,407\,962\,995\,501} - \frac{29\,975\,346\,986\,835\,504\,500\,518\,510\,615\,126\,088\,217\,584\,697\,791\,297\,v^4}{813\,922\,884\,566\,757\,179\,430\,511\,694\,216\,696\,156\,303\,518\,941\,513} -$$
$$\frac{70\,277\,972\,325\,846\,639\,061\,647\,195\,248\,910\,565\,060\,491\,883\,530\,157\,v^6}{1\,899\,153\,397\,322\,433\,418\,671\,193\,953\,172\,291\,031\,374\,877\,530\,197} - \frac{1\,926\,754\,541\,342\,822\,399\,506\,044\,786\,570\,961\,456\,182\,049\,757\,392\,v^8}{211\,017\,044\,146\,937\,046\,519\,021\,550\,352\,476\,781\,263\,875\,281\,133} +$$

$$\frac{100254295056986395896712880824527497685097738156994 v^{10}}{5697460191967300256013581859516873094124632590591} + \frac{10474992905716634957015163812916218183952303668147 v^{12}}{1899153397322433418671193953172291031374877530197} -$$

$$\frac{131223078764485655637458742083130155534080857895702 v^{14}}{24688994165191634442725521391239783407873407892561} - \frac{90282172472086282646716765586197154444226953214913 v^{16}}{137552967491781963323756476322621650415294701115697} +$$

$$\frac{367430753920217427452300911303599711593797994562727 v^{18}}{320956924147491247755431778086117184302354302603293} - \frac{68711002191812459345625738364226615213809888606810 v^{20}}{320956924147491247755431778086117184302354302603293} -$$

$$\frac{2513157577164488573825669024791521170943852438523 v^{22}}{35661880460832360861714642009568576033594922511477} + \frac{542526321836043091266409368775927169946056058706348 v^{24}}{12517320041752158662461839345358570187791817801528427} -$$

$$\frac{102926763075204167680376625240237378908442646170818 v^{26}}{12517320041752158662461839345358570187791817801528427} - \frac{53256571004991995940131510646791012033971012018447 v^{28}}{162725160542778062612003911489661412441293631419869551} +$$

$$\frac{30386283172849669931095948674643646995016180368909 v^{30}}{54241720180926020870667970496553804147097877139956517} - \frac{8400691503632549339556119511344124718138324811507 v^{32}}{54241720180926020870667970496553804147097877139956517} +$$

$$\frac{4122981123186925834204362434020505346464931187354 v^{34}}{162725160542778062612003911489661412441293631419869551} - \frac{149786196759253697556216809628657522169451451134 v^{36}}{54241720180926020870667970496553804147097877139956517} +$$

$$\frac{10431838957202624878839744576476004701395942156 v^{38}}{54241720180926020870667970496553804147097877139956517} - \frac{1101163842121353171371676670108916397406672160 v^{40}}{162725160542778062612003911489661412441293631419869551},$$

$$\frac{96632638432453628071050796796660223873313711316}{62609452658981321494654745708976627407962995501} - \frac{v}{2} + \frac{3637017704098182974627003435602279693868195305399 v^{2}}{876532337225738500925166439925672783711481937014} + \frac{5011928958072860605104979990835090507297547003227 v^{4}}{876532337225738500925166439925672783711481937014} +$$

$$\frac{1376615098014096136938460630402520413819930988119 v^{6}}{292177445741912833641722146641890927903827312338} + \frac{240670522744669421572142229851572198924328653252 v^{8}}{70339014715645682173007183450825593754625093711} - \frac{321815802559772789751607027647991936332144442343 v^{10}}{125218905317962642989309491417953254815925991002} -$$

$$\frac{45566706873078835901734848929076638452747688724532 v^{12}}{24688994165191634442725521391239783407873407892561} + \frac{24506886629878134026012747229041119152076506918488 v^{14}}{24688994165191634442725521391239783407873407892561} +$$

$$\frac{6665510819287368992592916284378586018031708720209 v^{16}}{21161994998735686665193304049634100063891492479338} - \frac{13782094473610302613153541442338082951083948881183 v^{18}}{49377988330383268885451042782479566815746815785122} + \frac{22831089135151560353745064161417834647761128928273 v^{20}}{641913848294982495510863556172234368604708605206586} +$$

$$\frac{1577717078663024528335174561058513334734442255241 v^{22}}{71323760921664721723429284019137152067189845022954} - \frac{270680287253828846123927649788599297302178483889791 v^{24}}{25034640083504317324923678690717140375583635603056854} +$$

$$\frac{22200136102960279235874120467635354497835586418433 v^{26}}{12517320041752158662461839345358570187791817801528427} + \frac{1966735258194424748954054506479683118080722563827 v^{28}}{12517320041752158662461839345358570187791817801528427} -$$

$$\frac{1238922646515362369789518418621953873382476290253 v^{30}}{8344880027834772441641226230239046791861211867685618} + \frac{321536629370674449080807747790435100057582471955 v^{32}}{8344880027834772441641226230239046791861211867685618} - \frac{153039971025005331660994874269660265473688335789 v^{34}}{25034640083504317324923678690717140375583635603056854} +$$

$$\frac{2720579755772758196222763279229524636919622656 v^{36}}{4172440013917386220820613115119523395930605933842809} - \frac{372108843164451880965088833131335627617449911 v^{38}}{8344880027834772441641226230239046791861211867685618} + \frac{19299124153388721784979223864064323408935722 v^{40}}{12517320041752158662461839345358570187791817801528427},$$

$$\frac{51232076726535592539693229608250643509094987669}{62609452658981321494654745708976627407962995501} + \frac{118369706842537629691237106836655411020037991001 v}{41739635105987547663103163805984418271975330334} + \frac{4521034244477742878475761118234091775535367947685 v^{2}}{876532337225738500925166439925672783711481937014} +$$

$$\frac{2267782116600307917175515388154910323317811842280 v^{3}}{146088722870956416820861073320945463951913656169} + \frac{50074058430284865739383297269331841317556332198586 v^{4}}{3067863180290084753238082539739854742990186779549} + \frac{652619836555520191863100587015747063755273115564275 v^{5}}{26588147562514067861396715344412074439248285422758} +$$

$$\frac{218548875672032974923105707620828368409511151042779 v^{6}}{13294073781257033930698357672206037219624142711379} + \frac{4786669672508467974962706573041071136854729361332 v^{7}}{340873686698898305915342504415539415887798531061} + \frac{7409024595306685097523365054508816364253435980699 v^{8}}{2954238618057118651266301704934674937694253935862} -$$

$$\frac{27046846608650996715154299123172993476801593561366 v^{9}}{4431357927085677976899452557402012406541380903793} - \frac{606364389221964438617207519649966389218905968457977 v^{10}}{79764442687542203584190146033236223317744856268274} -$$

$$\frac{2361654920514656224511692713685896097713376226114501 v^{11}}{345645918312682882198157299477356967710227710495854} - \frac{304143935131466726452959645611401720337024851192447 v^{12}}{172822959156341441099078649738678483855113855247927} +$$

$$\frac{26897140856140745443445924850404276930594864679193 v^{13}}{19202551017371271233230961082075387095012650583103} + \frac{373935027752140805340080281416135607206106156332104 v^{14}}{172822959156341441099078649738678483855113855247927} +$$

$$\frac{173675823646315297428036901487558339795074307613581 v^{15}}{115215306104227627399385766492452322570075903498618} + \frac{86000491925025361484128606585971079824977789107907 v^{16}}{518468877469024323297235949216035451565341565743781} -$$

$$\frac{1985557564253454143841313305653747926003888672611611 v^{17}}{4493396938064877468576044893205640580232960236446102} - \frac{1948832486639910979876523896350125788335495340653241 v^{18}}{4493396938064877468576044893205640580232960236446102} -$$

$$\frac{100861697423546965582808105655745835420484480173765 v^{19}}{748899489677479578096007482200940096705493372741017} + \frac{28787874908608959142994602174257656221981519003166 v^{20}}{320956924147491247755431778086117184302354302603293} +$$

$$\frac{2484098277488684167776872439376482375999932614879 v^{21}}{30567326109284880738612550293915922314509933581266} + \frac{2025067058009085173355344779640795776585795725605 v^{22}}{83211054408608842010667498022326677411721485860113} - \frac{47854316887612640578924437346397072123096298456695 v^{23}}{6490462243871489676832064845741480838114275897088814} -$$

$$\frac{1435890832156916893045687617953990174579838627441092 v^{24}}{87621240292265110637232875417509991314542724610698989} - \frac{141319342739124588412695304947358159876050506822456 v^{25}}{29207080097421703545744291805836663771514241536899663} +$$

$$\frac{21910692504062901037117032143351306134839165108649 v^{26}}{6740095407097316202864067339808460870349440354669153} + \frac{54624585236277236905181916369771636852387723859918 v^{27}}{29207080097421703545744291805836663771514241536899663} +$$

$$\frac{15134589278274004309660585062268415914574896840393 v^{28}}{175242480584530221274465750835019982629085449221397978} - \frac{82234510350145884918558677910991141953166060320682 v^{29}}{379692041266482146094675793475876629029685139979695619} -$$

$$\frac{6075883090121932734618739451043402466778348020592 v^{30}}{29207080097421703545744291805836663771514241536899663} - \frac{9294331922513437750654225423118168689447079276009 v^{31}}{253128027510988097396450528983917752686456759986463746} +$$

$$\frac{3434625279960877920644980706954304547140893683241 v^{32}}{58414160194843407091488583611673327543028483073799326} + \frac{177378002284448288270571475675298871699944119621 v^{33}}{9375112130036596199868538110515472321720620740239398} -$$

$$\frac{65505587152585247245890177540595025481092401157 v^{34}}{6740095407097316202864067339808460870349440354669153} - \frac{2872032655660774541933235757011080773023873761845 v^{35}}{759384082532964292189351586951753258059370279959391238} +$$

$$\frac{31\,159\,731\,172\,422\,683\,751\,836\,991\,618\,573\,032\,630\,350\,331\,941\,v^{36}}{29\,207\,080\,097\,421\,703\,545\,744\,291\,805\,836\,663\,771\,514\,241\,536\,899\,663}+\frac{19\,638\,934\,359\,829\,519\,971\,646\,697\,575\,360\,754\,952\,942\,941\,872\,v^{37}}{42\,188\,004\,585\,164\,682\,899\,408\,421\,497\,319\,625\,447\,742\,793\,331\,077\,291}-$$

$$\frac{89\,111\,499\,804\,706\,185\,823\,291\,165\,775\,440\,133\,449\,903\,219\,v^{38}}{1\,192\,125\,718\,262\,110\,348\,805\,889\,461\,462\,720\,970\,265\,887\,409\,669\,374}-\frac{1\,288\,445\,754\,125\,719\,819\,995\,901\,147\,839\,297\,283\,392\,038\,789\,v^{39}}{36\,161\,146\,787\,284\,013\,913\,778\,646\,997\,702\,536\,098\,065\,251\,426\,637\,678}+$$

$$\frac{463\,590\,092\,677\,173\,147\,225\,979\,672\,677\,982\,736\,237\,921\,771\,v^{40}}{175\,242\,480\,584\,530\,221\,274\,465\,750\,835\,019\,982\,629\,085\,449\,221\,397\,978}+\frac{5\,039\,624\,589\,011\,689\,115\,944\,760\,045\,941\,240\,123\,592\,935\,v^{41}}{3\,686\,330\,497\,732\,836\,369\,851\,221\,295\,882\,297\,369\,220\,244\,077\,472\,773},$$

$$-\frac{103\,744\,198\,520\,925\,139\,112\,697\,270\,483\,714\,205\,179\,239\,740\,775}{62\,609\,452\,658\,981\,321\,494\,654\,745\,708\,976\,627\,407\,962\,995\,501}+\frac{2\,946\,473\,470\,916\,063\,355\,182\,436\,215\,680\,337\,836\,653\,064\,387\,v}{1\,715\,327\,470\,109\,077\,301\,223\,417\,690\,656\,893\,901\,588\,027\,274}-\frac{169\,117\,028\,021\,158\,136\,798\,898\,697\,802\,651\,104\,449\,868\,270\,490\,v^{2}}{62\,609\,452\,658\,981\,321\,494\,654\,745\,708\,976\,627\,407\,962\,995\,501}+$$

$$\frac{3\,079\,821\,519\,870\,508\,742\,823\,591\,722\,468\,448\,475\,686\,037\,879\,892\,v^{3}}{438\,266\,168\,612\,869\,250\,462\,583\,219\,962\,836\,391\,855\,740\,968\,507}-\frac{144\,609\,526\,008\,548\,551\,881\,555\,944\,013\,721\,393\,879\,445\,611\,341\,670\,v^{4}}{39\,882\,221\,343\,771\,101\,792\,095\,073\,016\,618\,111\,658\,872\,428\,134\,137}+$$

$$\frac{1\,266\,281\,930\,426\,279\,961\,432\,726\,750\,106\,305\,838\,234\,883\,574\,378\,989\,v^{5}}{79\,764\,442\,687\,542\,203\,584\,190\,146\,033\,236\,223\,317\,744\,856\,268\,274}-\frac{35\,211\,959\,491\,211\,112\,438\,040\,482\,932\,956\,069\,526\,596\,418\,646\,644\,v^{6}}{13\,294\,073\,781\,257\,033\,930\,698\,357\,672\,206\,037\,219\,624\,142\,711\,379}+$$

$$\frac{17\,495\,094\,062\,130\,810\,447\,943\,886\,382\,786\,012\,602\,122\,436\,579\,255\,v^{7}}{1\,022\,621\,060\,096\,694\,917\,746\,027\,513\,246\,618\,247\,663\,395\,593\,183}-\frac{447\,767\,195\,687\,004\,889\,667\,891\,689\,142\,013\,169\,533\,432\,346\,171\,v^{8}}{328\,248\,735\,339\,679\,850\,140\,700\,189\,437\,186\,104\,188\,250\,437\,318}+\frac{29\,969\,535\,131\,228\,242\,752\,090\,842\,106\,104\,401\,444\,417\,273\,928\,752\,v^{9}}{4\,431\,357\,927\,085\,677\,976\,899\,452\,557\,402\,012\,406\,541\,380\,903\,793}+$$

$$\frac{54\,790\,495\,026\,817\,467\,205\,995\,515\,245\,022\,384\,433\,398\,905\,565\,755\,v^{10}}{39\,882\,221\,343\,771\,101\,792\,095\,073\,016\,618\,111\,658\,872\,428\,134\,137}-\frac{8\,637\,429\,237\,311\,416\,017\,223\,334\,511\,707\,984\,992\,638\,009\,734\,703\,155\,v^{11}}{1\,036\,937\,754\,938\,048\,646\,594\,471\,898\,432\,070\,903\,130\,683\,131\,487\,562}+$$

$$\frac{292\,997\,412\,065\,823\,374\,441\,827\,269\,254\,500\,524\,272\,857\,710\,726\,965\,v^{12}}{345\,645\,918\,312\,682\,882\,198\,157\,299\,477\,356\,967\,710\,227\,710\,495\,854}-\frac{1\,257\,974\,636\,010\,987\,044\,215\,894\,776\,752\,581\,174\,337\,038\,037\,901\,743\,v^{13}}{345\,645\,918\,312\,682\,882\,198\,157\,299\,477\,356\,967\,710\,227\,710\,495\,854}-$$

$$\frac{86\,202\,458\,485\,587\,948\,791\,063\,914\,728\,467\,896\,901\,358\,702\,126\,563\,v^{14}}{172\,822\,959\,156\,341\,441\,099\,078\,649\,738\,678\,483\,855\,113\,855\,247\,927}+\frac{937\,057\,183\,599\,568\,912\,356\,161\,343\,959\,395\,687\,905\,762\,684\,238\,243\,v^{15}}{345\,645\,918\,312\,682\,882\,198\,157\,299\,477\,356\,967\,710\,227\,710\,495\,854}-$$

$$\frac{1\,029\,058\,365\,402\,242\,798\,589\,954\,965\,330\,373\,400\,584\,250\,132\,023\,989\,v^{16}}{6\,740\,095\,407\,097\,316\,202\,864\,067\,339\,808\,460\,870\,349\,440\,354\,669\,153}+\frac{3\,481\,744\,270\,649\,927\,997\,745\,062\,944\,250\,461\,190\,852\,242\,319\,898\,760\,v^{17}}{6\,740\,095\,407\,097\,316\,202\,864\,067\,339\,808\,460\,870\,349\,440\,354\,669\,153}+$$

$$\frac{630\,987\,806\,296\,926\,525\,507\,389\,788\,477\,693\,355\,727\,548\,726\,901\,805\,v^{18}}{4\,493\,396\,938\,064\,877\,468\,576\,044\,893\,205\,640\,580\,232\,960\,236\,446\,102}-\frac{2\,886\,293\,540\,695\,871\,757\,609\,670\,588\,434\,371\,016\,013\,359\,348\,022\,501\,v^{19}}{4\,493\,396\,938\,064\,877\,468\,576\,044\,893\,205\,640\,580\,232\,960\,236\,446\,102}-$$

$$\frac{11\,695\,836\,760\,557\,019\,294\,108\,530\,145\,706\,531\,877\,914\,278\,327\,795\,v^{20}}{641\,913\,848\,294\,982\,495\,510\,863\,556\,172\,234\,368\,604\,708\,605\,206\,586}+\frac{33\,503\,685\,139\,373\,586\,506\,705\,203\,496\,244\,135\,433\,946\,752\,978\,175\,v^{21}}{320\,956\,924\,147\,491\,247\,755\,431\,778\,086\,117\,184\,302\,354\,302\,603\,293}-\frac{933\,719\,809\,757\,377\,120\,283\,101\,238\,618\,953\,301\,008\,150\,511\,776\,v^{22}}{83\,211\,054\,408\,608\,842\,010\,667\,498\,022\,326\,677\,411\,721\,485\,860\,113}+$$

$$\frac{285\,509\,658\,132\,130\,943\,523\,483\,897\,764\,494\,428\,305\,121\,616\,912\,451\,v^{23}}{6\,490\,462\,243\,871\,489\,676\,832\,064\,845\,741\,480\,838\,114\,275\,897\,088\,814}+\frac{74\,527\,647\,864\,487\,236\,161\,077\,169\,461\,283\,480\,050\,194\,940\,858\,945\,v^{24}}{13\,480\,190\,814\,194\,632\,405\,728\,134\,679\,616\,921\,740\,698\,880\,709\,338\,306}-$$

$$\frac{4\,281\,930\,517\,992\,655\,204\,598\,566\,032\,921\,897\,585\,481\,955\,956\,295\,731\,v^{25}}{175\,242\,480\,584\,530\,221\,274\,465\,750\,835\,019\,982\,629\,085\,449\,221\,397\,978}-\frac{79\,996\,284\,510\,325\,081\,252\,322\,072\,796\,183\,635\,058\,208\,989\,743\,605\,v^{26}}{87\,621\,240\,292\,265\,110\,637\,232\,875\,417\,509\,991\,314\,542\,724\,610\,698\,989}+$$

$$\frac{383\,238\,182\,461\,092\,841\,423\,418\,439\,322\,960\,861\,801\,297\,287\,608\,239\,v^{27}}{87\,621\,240\,292\,265\,110\,637\,232\,875\,417\,509\,991\,314\,542\,724\,610\,698\,989}-\frac{90\,949\,740\,287\,001\,694\,377\,152\,475\,730\,627\,325\,071\,184\,040\,706\,247\,v^{28}}{1\,139\,076\,123\,799\,446\,438\,284\,027\,380\,427\,629\,887\,089\,055\,419\,939\,086\,857}+$$

$$\frac{44\,496\,787\,411\,600\,803\,857\,789\,698\,432\,737\,956\,896\,878\,439\,073\,713\,v^{29}}{175\,242\,480\,584\,530\,221\,274\,465\,750\,835\,019\,982\,629\,085\,449\,221\,397\,978}+\frac{29\,005\,469\,483\,060\,268\,616\,630\,880\,549\,610\,368\,824\,464\,564\,183\,026\,v^{30}}{379\,692\,041\,266\,482\,146\,094\,675\,793\,475\,876\,629\,029\,685\,139\,979\,695\,619}-$$

$$\frac{258\,440\,829\,145\,137\,530\,404\,605\,903\,464\,123\,768\,026\,424\,713\,777\,v^{31}}{800\,193\,975\,271\,827\,494\,403\,953\,200\,159\,908\,596\,479\,842\,233\,887\,662}-\frac{15\,105\,121\,386\,794\,942\,457\,845\,417\,659\,926\,680\,191\,103\,349\,149\,489\,v^{32}}{759\,384\,082\,532\,964\,292\,189\,351\,586\,951\,753\,258\,059\,370\,279\,959\,391\,238}+\frac{5\,076\,489\,751\,874\,750\,855\,365\,658\,117\,271\,757\,568\,304\,642\,639\,599\,v^{33}}{58\,414\,160\,194\,843\,407\,091\,488\,583\,611\,673\,327\,543\,028\,483\,073\,799\,326}+$$

$$\frac{7\,206\,657\,958\,540\,817\,910\,830\,876\,529\,116\,665\,345\,460\,351\,836\,387\,v^{34}}{2\,278\,152\,247\,598\,892\,876\,568\,054\,760\,855\,259\,774\,178\,110\,839\,878\,173\,714}-\frac{2\,459\,062\,329\,592\,409\,364\,434\,257\,504\,888\,710\,895\,263\,653\,600\,449\,v^{35}}{175\,242\,480\,584\,530\,221\,274\,465\,750\,835\,019\,982\,629\,085\,449\,221\,397\,978}-$$

$$\frac{128\,397\,574\,359\,427\,943\,183\,393\,515\,751\,034\,838\,561\,159\,897\,960\,v^{36}}{379\,692\,041\,266\,482\,146\,094\,675\,793\,475\,876\,629\,029\,685\,139\,979\,695\,619}+\frac{44\,253\,473\,030\,732\,828\,961\,465\,804\,411\,387\,476\,590\,023\,299\,302\,v^{37}}{29\,207\,080\,097\,421\,703\,545\,744\,291\,805\,836\,663\,771\,514\,241\,536\,899\,663}+$$

$$\frac{1\,257\,361\,243\,081\,756\,241\,812\,953\,169\,898\,205\,300\,786\,049\,808\,v^{38}}{54\,241\,720\,180\,926\,020\,870\,667\,970\,496\,553\,804\,147\,097\,877\,139\,956\,517}-\frac{436\,768\,524\,450\,988\,420\,940\,093\,322\,243\,348\,554\,008\,744\,403\,v^{39}}{4\,172\,440\,013\,917\,386\,220\,820\,613\,115\,119\,523\,395\,930\,605\,933\,842\,809}-$$

$$\frac{1\,830\,964\,905\,870\,526\,079\,202\,217\,797\,311\,067\,649\,672\,579\,307\,v^{40}}{2\,278\,152\,247\,598\,892\,876\,568\,054\,760\,855\,259\,774\,178\,110\,839\,878\,173\,714}+\frac{320\,157\,006\,089\,788\,566\,467\,589\,826\,145\,131\,200\,974\,383\,336\,v^{41}}{87\,621\,240\,292\,265\,110\,637\,232\,875\,417\,509\,991\,314\,542\,724\,610\,698\,989},$$

$$-\frac{103\,744\,198\,520\,925\,139\,112\,697\,270\,483\,714\,205\,179\,239\,740\,775}{62\,609\,452\,658\,981\,321\,494\,654\,745\,708\,976\,627\,407\,962\,995\,501}-\frac{2\,946\,473\,470\,916\,063\,355\,182\,436\,215\,680\,337\,836\,653\,064\,387\,v}{1\,715\,327\,470\,109\,077\,301\,223\,417\,690\,656\,893\,901\,588\,027\,274}-\frac{169\,117\,028\,021\,158\,136\,798\,898\,697\,802\,651\,104\,449\,868\,270\,490\,v^{2}}{62\,609\,452\,658\,981\,321\,494\,654\,745\,708\,976\,627\,407\,962\,995\,501}-$$

$$\frac{3\,079\,821\,519\,870\,508\,742\,823\,591\,722\,468\,448\,475\,686\,037\,879\,892\,v^{3}}{438\,266\,168\,612\,869\,250\,462\,583\,219\,962\,836\,391\,855\,740\,968\,507}-\frac{144\,609\,526\,008\,548\,551\,881\,555\,944\,013\,721\,393\,879\,445\,611\,341\,670\,v^{4}}{39\,882\,221\,343\,771\,101\,792\,095\,073\,016\,618\,111\,658\,872\,428\,134\,137}-$$

$$\frac{1\,266\,281\,930\,426\,279\,961\,432\,726\,750\,106\,305\,838\,234\,883\,574\,378\,989\,v^{5}}{79\,764\,442\,687\,542\,203\,584\,190\,146\,033\,236\,223\,317\,744\,856\,268\,274}-\frac{35\,211\,959\,491\,211\,112\,438\,040\,482\,932\,956\,069\,526\,596\,418\,646\,644\,v^{6}}{13\,294\,073\,781\,257\,033\,930\,698\,357\,672\,206\,037\,219\,624\,142\,711\,379}-$$

$$\frac{17\,495\,094\,062\,130\,810\,447\,943\,886\,382\,786\,012\,602\,122\,436\,579\,255\,v^{7}}{1\,022\,621\,060\,096\,694\,917\,746\,027\,513\,246\,618\,247\,663\,395\,593\,183}-\frac{447\,767\,195\,687\,004\,889\,667\,891\,689\,142\,013\,169\,533\,432\,346\,171\,v^{8}}{328\,248\,735\,339\,679\,850\,140\,700\,189\,437\,186\,104\,188\,250\,437\,318}-\frac{29\,969\,535\,131\,228\,242\,752\,090\,842\,106\,104\,401\,444\,417\,273\,928\,752\,v^{9}}{4\,431\,357\,927\,085\,677\,976\,899\,452\,557\,402\,012\,406\,541\,380\,903\,793}+$$

$$\frac{54\,790\,495\,026\,817\,467\,205\,995\,515\,245\,022\,384\,433\,398\,905\,565\,755\,v^{10}}{39\,882\,221\,343\,771\,101\,792\,095\,073\,016\,618\,111\,658\,872\,428\,134\,137}+\frac{8\,637\,429\,237\,311\,416\,017\,223\,334\,511\,707\,984\,992\,638\,009\,734\,703\,155\,v^{11}}{1\,036\,937\,754\,938\,048\,646\,594\,471\,898\,432\,070\,903\,130\,683\,131\,487\,562}+$$

$$\frac{292\,997\,412\,065\,823\,374\,441\,827\,269\,254\,500\,524\,272\,857\,710\,726\,965\,v^{12}}{345\,645\,918\,312\,682\,882\,198\,157\,299\,477\,356\,967\,710\,227\,710\,495\,854}+\frac{1\,257\,974\,636\,010\,987\,044\,215\,894\,776\,752\,581\,174\,337\,038\,037\,901\,743\,v^{13}}{345\,645\,918\,312\,682\,882\,198\,157\,299\,477\,356\,967\,710\,227\,710\,495\,854}-$$

$$\frac{86\,202\,458\,485\,587\,948\,791\,063\,914\,728\,467\,896\,901\,358\,702\,126\,563\,v^{14}}{172\,822\,959\,156\,341\,441\,099\,078\,649\,738\,678\,483\,855\,113\,855\,247\,927}-\frac{937\,057\,183\,599\,568\,912\,356\,161\,343\,959\,395\,687\,905\,762\,684\,238\,243\,v^{15}}{345\,645\,918\,312\,682\,882\,198\,157\,299\,477\,356\,967\,710\,227\,710\,495\,854}-$$

$$\frac{1\,029\,058\,365\,402\,242\,798\,589\,954\,965\,330\,373\,400\,584\,250\,132\,023\,989\,v^{16}}{6\,740\,095\,407\,097\,316\,202\,864\,067\,339\,808\,460\,870\,349\,440\,354\,669\,153}-\frac{3\,481\,744\,270\,649\,927\,997\,745\,062\,944\,250\,461\,190\,852\,242\,319\,898\,760\,v^{17}}{6\,740\,095\,407\,097\,316\,202\,864\,067\,339\,808\,460\,870\,349\,440\,354\,669\,153}+$$

$$\frac{630\,987\,806\,296\,926\,525\,507\,389\,788\,477\,693\,355\,727\,548\,726\,901\,805\,v^{18}}{4\,493\,396\,938\,064\,877\,468\,576\,044\,893\,205\,640\,580\,232\,960\,236\,446\,102}+\frac{2\,886\,293\,540\,695\,871\,757\,609\,670\,588\,434\,371\,016\,013\,359\,348\,022\,501\,v^{19}}{4\,493\,396\,938\,064\,877\,468\,576\,044\,893\,205\,640\,580\,232\,960\,236\,446\,102}-$$

$$\frac{11\,695\,836\,760\,557\,019\,294\,108\,530\,145\,706\,531\,877\,914\,278\,327\,795\,v^{20}}{641\,913\,848\,294\,982\,495\,510\,863\,556\,172\,234\,368\,604\,708\,605\,206\,586}-\frac{33\,503\,685\,139\,373\,586\,506\,705\,203\,496\,244\,135\,433\,946\,752\,978\,175\,v^{21}}{320\,956\,924\,147\,491\,247\,755\,431\,778\,086\,117\,184\,302\,354\,302\,603\,293}-\frac{933\,719\,809\,757\,377\,120\,283\,101\,238\,618\,953\,301\,008\,150\,511\,776\,v^{22}}{83\,211\,054\,408\,608\,842\,010\,667\,498\,022\,326\,677\,411\,721\,485\,860\,113}-$$

$$\frac{285\,509\,658\,132\,130\,943\,523\,483\,897\,764\,494\,428\,305\,121\,616\,912\,451\,v^{23}}{6\,490\,462\,243\,871\,489\,676\,832\,064\,845\,741\,480\,838\,114\,275\,897\,088\,814}+\frac{74\,527\,647\,864\,487\,236\,161\,077\,169\,461\,283\,480\,050\,194\,940\,858\,945\,v^{24}}{13\,480\,190\,814\,194\,632\,405\,728\,134\,679\,616\,921\,740\,698\,880\,709\,338\,306}+$$

$$\frac{4\,281\,930\,517\,992\,655\,204\,598\,566\,032\,921\,897\,585\,481\,955\,956\,295\,731\,v^{25}}{175\,242\,480\,584\,530\,221\,274\,465\,750\,835\,019\,982\,629\,085\,449\,221\,397\,978}-\frac{79\,996\,284\,510\,325\,081\,252\,322\,072\,796\,183\,635\,058\,208\,989\,743\,605\,v^{26}}{87\,621\,240\,292\,265\,110\,637\,232\,875\,417\,509\,991\,314\,542\,724\,610\,698\,989}-$$

$$\frac{383\,238\,182\,461\,092\,841\,423\,418\,439\,322\,960\,861\,801\,297\,287\,608\,239\,v^{27}}{87\,621\,240\,292\,265\,110\,637\,232\,875\,417\,509\,991\,314\,542\,724\,610\,698\,989}-\frac{90\,949\,740\,287\,001\,694\,377\,152\,475\,730\,627\,325\,071\,184\,040\,706\,247\,v^{28}}{1\,139\,076\,123\,799\,446\,438\,284\,027\,380\,427\,629\,887\,089\,055\,419\,939\,086\,857}-$$

$$\frac{44\,496\,787\,411\,600\,803\,857\,789\,698\,432\,737\,956\,896\,878\,439\,073\,713\,v^{29}}{175\,242\,480\,584\,530\,221\,274\,465\,750\,835\,019\,982\,629\,085\,449\,221\,397\,978}+\frac{29\,005\,469\,483\,060\,268\,616\,630\,880\,549\,610\,368\,824\,464\,564\,183\,026\,v^{30}}{379\,692\,041\,266\,482\,146\,094\,675\,793\,475\,876\,629\,029\,685\,139\,979\,695\,619}+$$

$$\frac{258\,440\,829\,145\,137\,530\,404\,605\,903\,464\,123\,768\,026\,424\,713\,777\,v^{31}}{800\,193\,975\,271\,827\,494\,403\,953\,200\,159\,908\,596\,479\,842\,233\,887\,662}-\frac{15\,105\,121\,386\,794\,942\,457\,845\,417\,659\,926\,680\,191\,103\,349\,149\,489\,v^{32}}{759\,384\,082\,532\,964\,292\,189\,351\,586\,951\,753\,258\,059\,370\,279\,959\,391\,238}-\frac{5\,076\,489\,751\,874\,750\,855\,365\,658\,117\,271\,757\,568\,304\,642\,639\,599\,v^{33}}{58\,414\,160\,194\,843\,407\,091\,488\,583\,611\,673\,327\,543\,028\,483\,073\,799\,326}+$$

$$\frac{7\,206\,657\,958\,540\,817\,910\,830\,876\,529\,116\,665\,345\,460\,351\,836\,387\,v^{34}}{2\,278\,152\,247\,598\,892\,876\,568\,054\,760\,855\,259\,774\,178\,110\,839\,878\,173\,714}+\frac{2\,459\,062\,329\,592\,409\,364\,434\,257\,504\,888\,710\,895\,263\,653\,600\,449\,v^{35}}{175\,242\,480\,584\,530\,221\,274\,465\,750\,835\,019\,982\,629\,085\,449\,221\,397\,978}-$$

$$\frac{128\,397\,574\,359\,427\,943\,183\,393\,515\,751\,034\,838\,561\,159\,897\,960\,v^{36}}{379\,692\,041\,266\,482\,146\,094\,675\,793\,475\,876\,629\,029\,685\,139\,979\,695\,619}-\frac{44\,253\,473\,030\,732\,828\,961\,465\,804\,411\,387\,476\,590\,023\,299\,302\,v^{37}}{29\,207\,080\,097\,421\,703\,545\,744\,291\,805\,836\,663\,771\,514\,241\,536\,899\,663}+$$

$$\frac{1\,257\,361\,243\,081\,756\,241\,812\,953\,169\,898\,205\,300\,786\,049\,808\,v^{38}}{54\,241\,720\,180\,926\,020\,870\,667\,970\,496\,553\,804\,147\,097\,877\,139\,956\,517}+\frac{436\,768\,524\,450\,988\,420\,940\,093\,322\,243\,348\,554\,008\,744\,403\,v^{39}}{4\,172\,440\,013\,917\,386\,220\,820\,613\,115\,119\,523\,395\,930\,605\,933\,842\,809}-$$

$$\frac{1\,830\,964\,905\,870\,526\,079\,202\,217\,797\,311\,067\,649\,672\,579\,307\,v^{40}}{2\,278\,152\,247\,598\,892\,876\,568\,054\,760\,855\,259\,774\,178\,110\,839\,878\,173\,714}-\frac{320\,157\,006\,089\,788\,566\,467\,589\,826\,145\,131\,200\,974\,383\,336\,v^{41}}{87\,621\,240\,292\,265\,110\,637\,232\,875\,417\,509\,991\,314\,542\,724\,610\,698\,989},$$

$$\frac{51\,232\,076\,726\,535\,592\,539\,693\,229\,608\,250\,643\,509\,094\,987\,669}{62\,609\,452\,658\,981\,321\,494\,654\,745\,708\,976\,627\,407\,962\,995\,501}-\frac{118\,369\,706\,842\,537\,629\,691\,237\,106\,836\,655\,411\,020\,037\,991\,001\,v}{41\,739\,635\,105\,987\,547\,663\,103\,163\,805\,984\,418\,271\,975\,330\,334}+\frac{4\,521\,034\,244\,477\,742\,878\,475\,761\,118\,234\,091\,775\,535\,367\,947\,685\,v^{2}}{876\,532\,337\,225\,738\,500\,925\,166\,439\,925\,672\,783\,711\,481\,937\,014}-$$

$$\frac{2\,267\,782\,116\,600\,307\,917\,175\,515\,388\,154\,910\,323\,317\,811\,842\,280\,v^{3}}{146\,088\,722\,870\,956\,416\,820\,861\,073\,320\,945\,463\,951\,913\,656\,169}+\frac{50\,074\,058\,430\,284\,865\,739\,383\,297\,269\,331\,841\,317\,556\,332\,198\,586\,v^{4}}{3\,067\,863\,180\,290\,084\,753\,238\,082\,539\,739\,854\,742\,990\,186\,779\,549}-\frac{652\,619\,836\,555\,520\,191\,863\,100\,587\,015\,747\,063\,755\,273\,115\,564\,275\,v^{5}}{26\,588\,147\,562\,514\,067\,861\,396\,715\,344\,412\,074\,439\,248\,285\,422\,758}+$$

$$\frac{218\,548\,875\,672\,032\,974\,923\,105\,707\,620\,828\,368\,409\,511\,151\,042\,779\,v^{6}}{13\,294\,073\,781\,257\,033\,930\,698\,357\,672\,206\,037\,219\,624\,142\,711\,379}-\frac{4\,786\,669\,672\,508\,467\,974\,962\,706\,573\,041\,071\,136\,854\,729\,361\,332\,v^{7}}{340\,873\,686\,698\,898\,305\,915\,342\,504\,415\,539\,415\,887\,798\,531\,061}+\frac{7\,409\,024\,595\,306\,685\,097\,523\,365\,054\,508\,816\,364\,253\,435\,980\,699\,v^{8}}{2\,954\,238\,618\,057\,118\,651\,266\,301\,704\,934\,674\,937\,694\,253\,935\,862}+$$

$$\frac{27\,046\,846\,608\,650\,996\,715\,154\,299\,123\,172\,993\,476\,801\,593\,561\,366\,v^{9}}{4\,431\,357\,927\,085\,677\,976\,899\,452\,557\,402\,012\,406\,541\,380\,903\,793}-\frac{606\,364\,389\,221\,964\,438\,617\,207\,519\,649\,966\,389\,218\,905\,968\,457\,977\,v^{10}}{79\,764\,442\,687\,542\,203\,584\,190\,146\,033\,236\,223\,317\,744\,856\,268\,274}+$$

$$\frac{2\,361\,654\,920\,514\,656\,224\,511\,692\,713\,685\,896\,097\,713\,376\,226\,114\,501\,v^{11}}{345\,645\,918\,312\,682\,882\,198\,157\,299\,477\,356\,967\,710\,227\,710\,495\,854}-\frac{304\,143\,935\,131\,466\,726\,452\,959\,645\,611\,401\,720\,337\,024\,851\,192\,447\,v^{12}}{172\,822\,959\,156\,341\,441\,099\,078\,649\,738\,678\,483\,855\,113\,855\,247\,927}-$$

$$\frac{26\,897\,140\,856\,140\,745\,443\,445\,924\,850\,404\,276\,930\,594\,864\,679\,193\,v^{13}}{19\,202\,551\,017\,371\,271\,233\,230\,961\,082\,075\,387\,095\,012\,650\,583\,103}+\frac{373\,935\,027\,752\,140\,805\,340\,080\,281\,416\,135\,607\,206\,106\,156\,332\,104\,v^{14}}{172\,822\,959\,156\,341\,441\,099\,078\,649\,738\,678\,483\,855\,113\,855\,247\,927}-$$

$$\frac{173\,675\,823\,646\,315\,297\,428\,036\,901\,487\,558\,339\,795\,074\,307\,613\,581\,v^{15}}{115\,215\,306\,104\,227\,627\,399\,385\,766\,492\,452\,322\,570\,075\,903\,498\,618}+\frac{86\,000\,491\,925\,025\,361\,484\,128\,606\,585\,971\,079\,824\,977\,789\,107\,907\,v^{16}}{518\,468\,877\,469\,024\,323\,297\,235\,949\,216\,035\,451\,565\,341\,565\,743\,781}+$$

$$\frac{1\,985\,557\,564\,253\,454\,143\,841\,313\,305\,653\,747\,926\,003\,888\,672\,611\,611\,v^{17}}{4\,493\,396\,938\,064\,877\,468\,576\,044\,893\,205\,640\,580\,232\,960\,236\,446\,102}-\frac{1\,948\,832\,486\,639\,910\,979\,876\,523\,896\,350\,125\,788\,335\,495\,340\,653\,241\,v^{18}}{4\,493\,396\,938\,064\,877\,468\,576\,044\,893\,205\,640\,580\,232\,960\,236\,446\,102}+$$

$$\frac{100\,861\,697\,423\,546\,965\,582\,808\,105\,655\,745\,835\,420\,484\,480\,173\,765\,v^{19}}{748\,899\,489\,677\,479\,578\,096\,007\,482\,200\,940\,096\,705\,493\,372\,741\,017}+\frac{28\,787\,874\,908\,608\,959\,142\,994\,602\,174\,257\,656\,221\,981\,519\,003\,166\,v^{20}}{320\,956\,924\,147\,491\,247\,755\,431\,778\,086\,117\,184\,302\,354\,302\,603\,293}-$$

$$\frac{2\,484\,098\,277\,488\,684\,167\,776\,872\,439\,376\,482\,375\,999\,932\,614\,879\,v^{21}}{30\,567\,326\,109\,284\,880\,738\,612\,550\,293\,915\,922\,314\,509\,933\,581\,266}+\frac{2\,025\,067\,058\,009\,085\,173\,355\,344\,779\,640\,795\,776\,585\,795\,725\,605\,v^{22}}{83\,211\,054\,408\,608\,842\,010\,667\,498\,022\,326\,677\,411\,721\,485\,860\,113}+\frac{47\,854\,316\,887\,612\,640\,578\,924\,437\,346\,397\,072\,123\,096\,298\,456\,695\,v^{23}}{6\,490\,462\,243\,871\,489\,676\,832\,064\,845\,741\,480\,838\,114\,275\,897\,088\,814}-$$

$$\frac{1\,435\,890\,832\,156\,916\,893\,045\,687\,617\,953\,990\,174\,579\,838\,627\,441\,092\,v^{24}}{87\,621\,240\,292\,265\,110\,637\,232\,875\,417\,509\,991\,314\,542\,724\,610\,698\,989}+\frac{141\,319\,342\,739\,124\,588\,412\,695\,304\,947\,358\,159\,876\,050\,506\,822\,456\,v^{25}}{29\,207\,080\,097\,421\,703\,545\,744\,291\,805\,836\,663\,771\,514\,241\,536\,899\,663}+$$

$$\frac{21\,910\,692\,504\,062\,901\,037\,117\,032\,143\,351\,306\,134\,839\,165\,108\,649\,v^{26}}{6\,740\,095\,407\,097\,316\,202\,864\,067\,339\,808\,460\,870\,349\,440\,354\,669\,153}-\frac{54\,624\,585\,236\,277\,236\,905\,181\,916\,369\,771\,636\,852\,387\,723\,859\,918\,v^{27}}{29\,207\,080\,097\,421\,703\,545\,744\,291\,805\,836\,663\,771\,514\,241\,536\,899\,663}+$$

$$\frac{15\,134\,589\,278\,274\,004\,309\,660\,585\,062\,268\,415\,914\,574\,896\,840\,393\,v^{28}}{175\,242\,480\,584\,530\,221\,274\,465\,750\,835\,019\,982\,629\,085\,449\,221\,397\,978}+\frac{82\,234\,510\,350\,145\,884\,918\,558\,677\,910\,991\,141\,953\,166\,060\,320\,682\,v^{29}}{379\,692\,041\,266\,482\,146\,094\,675\,793\,475\,876\,629\,029\,685\,139\,979\,695\,619}-$$

$$\frac{6\,075\,883\,090\,121\,932\,734\,618\,739\,451\,043\,402\,466\,778\,348\,020\,592\,v^{30}}{29\,207\,080\,097\,421\,703\,545\,744\,291\,805\,836\,663\,771\,514\,241\,536\,899\,663}+\frac{9\,294\,331\,922\,513\,437\,750\,654\,225\,423\,118\,168\,689\,447\,079\,276\,009\,v^{31}}{253\,128\,027\,510\,988\,097\,396\,450\,528\,983\,917\,752\,686\,456\,759\,986\,463\,746}+$$

$$\frac{3\,434\,625\,279\,960\,877\,920\,644\,980\,706\,954\,304\,547\,140\,893\,683\,241\,v^{32}}{58\,414\,160\,194\,843\,407\,091\,488\,583\,611\,673\,327\,543\,028\,483\,073\,799\,326}-\frac{177\,378\,002\,284\,448\,288\,270\,571\,475\,675\,298\,871\,699\,944\,119\,621\,v^{33}}{9\,375\,112\,130\,036\,596\,199\,868\,538\,110\,515\,472\,321\,720\,620\,740\,239\,398}-$$

$$\frac{65\,505\,587\,152\,585\,247\,245\,890\,177\,540\,595\,025\,481\,092\,401\,157\,v^{34}}{6\,740\,095\,407\,097\,316\,202\,864\,067\,339\,808\,460\,870\,349\,440\,354\,669\,153}+\frac{2\,872\,032\,655\,660\,774\,541\,933\,235\,757\,011\,080\,773\,023\,873\,761\,845\,v^{35}}{759\,384\,082\,532\,964\,292\,189\,351\,586\,951\,753\,258\,059\,370\,279\,959\,391\,238}+$$

$$\frac{31\,159\,731\,172\,422\,683\,751\,836\,991\,618\,573\,032\,630\,350\,331\,941\,v^{36}}{29\,207\,080\,097\,421\,703\,545\,744\,291\,805\,836\,663\,771\,514\,241\,536\,899\,663}-\frac{19\,638\,934\,359\,829\,519\,971\,646\,697\,575\,360\,754\,952\,942\,941\,872\,v^{37}}{42\,188\,004\,585\,164\,682\,899\,408\,421\,497\,319\,625\,447\,742\,793\,331\,077\,291}-$$

$$\frac{89\,111\,499\,804\,706\,185\,823\,291\,165\,775\,440\,133\,449\,903\,219\,v^{38}}{1\,192\,125\,718\,262\,110\,348\,805\,889\,461\,462\,720\,970\,265\,887\,409\,669\,374}+\frac{1\,288\,445\,754\,125\,719\,819\,995\,901\,147\,839\,297\,283\,392\,038\,789\,v^{39}}{36\,161\,146\,787\,284\,013\,913\,778\,646\,997\,702\,536\,098\,065\,251\,426\,637\,678}+$$

$$\frac{463\,590\,092\,677\,173\,147\,225\,979\,672\,677\,982\,736\,237\,921\,771\,v^{40}}{175\,242\,480\,584\,530\,221\,274\,465\,750\,835\,019\,982\,629\,085\,449\,221\,397\,978}-\frac{5\,039\,624\,589\,011\,689\,115\,944\,760\,045\,941\,240\,123\,592\,935\,v^{41}}{3\,686\,330\,497\,732\,836\,369\,851\,221\,295\,882\,297\,369\,220\,244\,077\,472\,773}\};$$

今回は 参考文献[1]「可解な 5 次方程式について」を「参考」にして， Gal= F_{42}の方程式を解いてみたいと思います． しかし， 5次方程式の場合(Another_F20.nb)とは異なり， 一筋縄では行きません．

§１.6次の補助方程式 $h(x)$ を作る

1-1. 補助方程式の定義

$C_7 \triangleright \{\mathrm{Id}\}$の部分に注目し， ラグランジュの7次分解式をつくる .(ζは1の原始7乗根のひとつ．) このとき x_k（$1 \leqq k \leqq 7$）を自由変数， すなわち $f = 0$ の解となっていることは忘れるものとする．

In[ ]:=
```
Clear[x1, x2, x3, x4, x5, x6, x7, ξ]
r0 = x1 + x2 + x3 + x4 + x5 + x6 + x7;
r1 = x1 + ξ x2 + ξ^2 x3 + ξ^3 x4 + ξ^4 x5 + ξ^5 x6 + ξ^6 x7;
r2 = x1 + ξ^2 x2 + ξ^4 x3 + ξ^6 x4 + ξ x5 + ξ^3 x6 + ξ^5 x7;
r3 = x1 + ξ^3 x2 + ξ^6 x3 + ξ^2 x4 + ξ^5 x5 + ξ x6 + ξ^4 x7;
r4 = x1 + ξ^4 x2 + ξ x3 + ξ^5 x4 + ξ^2 x5 + ξ^6 x6 + ξ^3 x7;
r5 = x1 + ξ^5 x2 + ξ^3 x3 + ξ x4 + ξ^6 x5 + ξ^4 x6 + ξ^2 x7;
r6 = x1 + ξ^6 x2 + ξ^5 x3 + ξ^4 x4 + ξ^3 x5 + ξ^2 x6 + ξ x7;
```

解と係数の関係より r0 = 0. またσ=(1,2,3,4,5,6,7),τ=(1,3,2,6,4,5)とおくと, F_{42}=<σ,τ>

$$
(\#)\begin{cases}
\text{r1を}\sigma\text{で移すと r1} \xrightarrow{\sigma} \zeta^6\,\text{r1}, & \tau\text{で移すと r1} \xrightarrow{\tau} \zeta^4\,\text{r5} \\
\text{r2を}\sigma\text{で移すと r2} \xrightarrow{\sigma} \zeta^5\,\text{r2}, & \tau\text{で移すと r2} \xrightarrow{\tau} \zeta\,\text{r3} \\
\text{r3を}\sigma\text{で移すと r3} \xrightarrow{\sigma} \zeta^4\,\text{r3}, & \tau\text{で移すと r3} \xrightarrow{\tau} \zeta^5\,\text{r1} \\
\text{r4を}\sigma\text{で移すと r4} \xrightarrow{\sigma} \zeta^3\,\text{r4}, & \tau\text{で移すと r4} \xrightarrow{\tau} \zeta^2\,\text{r6} \\
\text{r5を}\sigma\text{で移すと r5} \xrightarrow{\sigma} \zeta^2\,\text{r5}, & \tau\text{で移すと r5} \xrightarrow{\tau} \zeta^6\,\text{r4} \\
\text{r6を}\sigma\text{で移すと r6} \xrightarrow{\sigma} \zeta\,\text{r6}, & \tau\text{で移すと r6} \xrightarrow{\tau} \zeta^3\,\text{r2}
\end{cases}
$$

即ち, σは 分解式を「r1 $\to \zeta^6$ r1 $\to \zeta^5$ r1 $\to \zeta^4$ r1 $\to \zeta^3$ r1 $\to \to \zeta^2$ r1 $\to \zeta$ r1 $\to$ r1」のように移し、τ は「r1 $\to \zeta^4$ r5 $\to \zeta^3$ r4 $\to \zeta^5$ r6 $\to \zeta$ r2 $\to \zeta^2$ r3 $\to$ r1」のように移す.

さらにfの最小分解体K内の 同型写像 $\omega: \zeta \to \zeta^4$ とすると， 分解式は次のように移る． ωは, r_k についてはτと同様に働く.

(by ω) r1→r5→r4→r6 →r2→r3 →r1…

$R_1 = {r_1}^7$, $R_2 = {r_2}^7$, $R_3 = {r_3}^7$, $R_4 = {r_4}^7$, $R_5 = {r_5}^7$, $R_6 = {r_6}^7$ と定義し， ${r_1}^7$ を展開し, ζについて整理した式を $R_1 = {r_1}^7 = l_0 + l_1\,\zeta + l_2\,\zeta^2 + l_3\,\zeta^3 + l_4\,\zeta^4 + l_5\,\zeta^5 + l_6\,\zeta^6$ (l_i は ζを含まない x_j のみの式)とおく.

これを ω で移すと次の式が得られる.

$$
(\#2)\begin{cases}
R_1 = {r_1}^7 = l_0 + l_1\,\zeta + l_2\,\zeta^2 + l_3\,\zeta^3 + l_4\,\zeta^4 + l_5\,\zeta^5 + l_6\,\zeta^6 \\
R_2 = {r_2}^7 = l_0 + l_4\,\zeta + l_1\,\zeta^2 + l_5\,\zeta^3 + l_2\,\zeta^4 + l_6\,\zeta^5 + l_3\,\zeta^6 \\
R_3 = {r_3}^7 = l_0 + l_5\,\zeta + l_3\,\zeta^2 + l_1\,\zeta^3 + l_6\,\zeta^4 + l_4\,\zeta^5 + l_2\,\zeta^6 \\
R_4 = {r_4}^7 = l_0 + l_2\,\zeta + l_4\,\zeta^2 + l_6\,\zeta^3 + l_1\,\zeta^4 + l_3\,\zeta^5 + l_5\,\zeta^6 \\
R_5 = {r_5}^7 = l_0 + l_3\,\zeta + l_6\,\zeta^2 + l_2\,\zeta^3 + l_5\,\zeta^4 + l_1\,\zeta^5 + l_4\,\zeta^6 \\
R_6 = {r_6}^7 = l_0 + l_6\,\zeta + l_5\,\zeta^2 + l_4\,\zeta^3 + l_3\,\zeta^4 + l_2\,\zeta^5 + l_1\,\zeta^6
\end{cases}
$$

(♯1) より，σで $R_1 \sim R_6$ は動かない．よってσで $l_0 \sim l_6$ も動かない．
 次に τによって「$R_1 \to R_5 \to R_4 \to R_6 \to R_2 \to R_3 \to R_1$」と動くから，
 「$l_1 \to l_3 \to l_2 \to l_6 \to l_4 \to l_5 \to l_1$」と動く．また l_0 は動かない．

次に $\boldsymbol{h(x) = (x - l_1)(x - l_2)(x - l_3)(x - l_4)(x - l_5)(x - l_6)}$ と定義すると，
 l_i $(1 \leqq i \leqq 6)$ は τによって動くので有理数でない．

また σ, τ によって $h(x)$ の係数は不変なので，$h(x)$ の係数は $F_{42} = \langle \sigma, \tau \rangle$ の固定体Q(ξ) に入る．しかし，l_i $(1 \leqq i \leqq 6)$ は ξ を含んでないから
$h(x)$ の係数は有理数となる．

まとめると「**$h(x)$ は Q上既約な 有理数係数の6次式**」となる．
 同様に l_0 も σとτによって動かず ξ を含んでないから 有理数となる．

1-2. 補助方程式h(x)の式を求める

以上を Mathematica で実行しましょう．解くだけなら不要ですが，
 先ず l_i $(0 \leqq i \leqq 6)$ を $x_1 \sim x_7$ の式で見てみましょう．

In[ ]:=
```
r1^7 // PolynomialMod[#, ξ^7 - 1] & // Collect[#, ξ] &
```

Out[ ]=
```
x1^7 + x2^7 + … 248 … + ( … 1 … ) ξ^5 +
 (7 x1 x2^6 + 105 x1^2 x2^4 x3 + 210 x1^3 x2^2 x3^2 + 35 x1^4 x3^3 + 7 x2 x3^6 + 140 x1^3 x2^3 x4 + 210 x1^4 x2 x3 x4 + 105 x2^2 x3^4 x4 +
   42 x1 x3^5 x4 + 21 x1^5 x4^2 + 210 x2^3 x3^2 x4^2 + 420 x1 x2 x3^3 x4^2 + 35 x2^4 x4^3 + 420 x1 x2^2 x3 x4^3 + 210 x1^2 x3^2 x4^3 +
   105 x1^2 x2 x4^4 + 7 x3 x4^6 + 105 x1^4 x2^2 x5 + 42 x1^5 x3 x5 + 140 x2^3 x3^3 x5 + 210 x1 x2 x3^4 x5 + 210 x2^4 x3 x4 x5 +
   1260 x1 x2^2 x3^2 x4 x5 + 420 x1^2 x3^3 x4 x5 + 420 x1 x2^3 x4^2 x5 + 1260 x1^2 x2 x3 x4^2 x5 + 140 x1^3 x4^3 x5 + 105 x3^2 x4^4 x5 +
   42 x2 x4^5 x5 + 21 x2^5 x5^2 + 420 x1 x2^3 x3 x5^2 + 630 x1^2 x2 x3^2 x5^2 + 630 x1^2 x2^2 x4 x5^2 + … 179 … + 210 x1^2 x6^3 x7^2 +
   420 x4 x5 x6^3 x7^2 + 105 x3 x6^4 x7^2 + 210 x1^2 x2^2 x7^3 + 140 x1^3 x3 x7^3 + 140 x3^3 x4 x7^3 + 420 x2 x3 x4^2 x7^3 +
   140 x1 x4^3 x7^3 + 420 x2 x3^2 x5 x7^3 + 420 x2^2 x4 x5 x7^3 + 840 x1 x3 x4 x5 x7^3 + 420 x1 x2 x5^2 x7^3 + 35 x5^4 x7^3 +
   420 x2^2 x3 x6 x7^3 + 420 x1 x3^2 x6 x7^3 + 840 x1 x2 x4 x6 x7^3 + 420 x1^2 x5 x6 x7^3 + 420 x4 x5^2 x6 x7^3 + 210 x4^2 x6^2 x7^3 +
   420 x3 x5 x6^2 x7^3 + 140 x2 x6^3 x7^3 + 35 x2^3 x7^4 + 210 x1 x2 x3 x7^4 + 105 x1^2 x4 x7^4 + 105 x4^2 x5 x7^4 + 105 x3 x5^2 x7^4 +
   210 x3 x4 x6 x7^4 + 210 x2 x5 x6 x7^4 + 105 x1 x6^2 x7^4 + 21 x3^2 x7^5 + 42 x2 x4 x7^5 + 42 x1 x5 x7^5 + 7 x6 x7^6) ξ^6
```

完全な式は使えません（もとのメモリサイズ：409.1 kB）

全てが長いので l_0 だけ出力します．

In[ ]:=
```
{l0, l1, l2, l3, l4, l5, l6} = CoefficientList[%, ζ]; l0
```

Out[ ]=

$$
\begin{aligned}
&x1^{7} + x2^{7} + 42\,x1\,x2^{5}\,x3 + 210\,x1^{2}\,x2^{3}\,x3^{2} + 140\,x1^{3}\,x2\,x3^{3} + x3^{7} + 105\,x1^{2}\,x2^{4}\,x4 + 420\,x1^{3}\,x2^{2}\,x3\,x4 + \\
&105\,x1^{4}\,x3^{2}\,x4 + 42\,x2\,x3^{5}\,x4 + 105\,x1^{4}\,x2\,x4^{2} + 210\,x2^{2}\,x3^{3}\,x4^{2} + 105\,x1\,x3^{4}\,x4^{2} + 140\,x2^{3}\,x3\,x4^{3} + \\
&420\,x1\,x2\,x3^{2}\,x4^{3} + 105\,x1\,x2^{2}\,x4^{4} + 105\,x1^{2}\,x3\,x4^{4} + x4^{7} + 140\,x1^{3}\,x2^{3}\,x5 + 210\,x1^{4}\,x2\,x3\,x5 + \\
&105\,x2^{2}\,x3^{4}\,x5 + 42\,x1\,x3^{5}\,x5 + 42\,x1^{5}\,x4\,x5 + 420\,x2^{3}\,x3^{2}\,x4\,x5 + 840\,x1\,x2\,x3^{3}\,x4\,x5 + 105\,x2^{4}\,x4^{2}\,x5 + \\
&1260\,x1\,x2^{2}\,x3\,x4^{2}\,x5 + 630\,x1^{2}\,x3^{2}\,x4^{2}\,x5 + 420\,x1^{2}\,x2\,x4^{3}\,x5 + 42\,x3\,x4^{5}\,x5 + 105\,x2^{4}\,x3\,x5^{2} + \\
&630\,x1\,x2^{2}\,x3^{2}\,x5^{2} + 210\,x1^{2}\,x3^{3}\,x5^{2} + 420\,x1\,x2^{3}\,x4\,x5^{2} + 1260\,x1^{2}\,x2\,x3\,x4\,x5^{2} + 210\,x1^{3}\,x4^{2}\,x5^{2} + \\
&210\,x3^{2}\,x4^{3}\,x5^{2} + 105\,x2\,x4^{4}\,x5^{2} + 210\,x1^{2}\,x2^{2}\,x5^{3} + 140\,x1^{3}\,x3\,x5^{3} + 140\,x3^{3}\,x4\,x5^{3} + 420\,x2\,x3\,x4^{2}\,x5^{3} + \\
&140\,x1\,x4^{3}\,x5^{3} + 105\,x2\,x3^{2}\,x5^{4} + 105\,x2^{2}\,x4\,x5^{4} + 210\,x1\,x3\,x4\,x5^{4} + 42\,x1\,x2\,x5^{5} + x5^{7} + 105\,x1^{4}\,x2^{2}\,x6 + \\
&42\,x1^{5}\,x3\,x6 + 140\,x2^{3}\,x3^{3}\,x6 + 210\,x1\,x2\,x3^{4}\,x6 + 210\,x2^{4}\,x3\,x4\,x6 + 1260\,x1\,x2^{2}\,x3^{2}\,x4\,x6 + \\
&420\,x1^{2}\,x3^{3}\,x4\,x6 + 420\,x1\,x2^{3}\,x4^{2}\,x6 + 1260\,x1^{2}\,x2\,x3\,x4^{2}\,x6 + 140\,x1^{3}\,x4^{3}\,x6 + 105\,x3^{2}\,x4^{4}\,x6 + \\
&42\,x2\,x4^{5}\,x6 + 42\,x2^{5}\,x5\,x6 + 840\,x1\,x2^{3}\,x3\,x5\,x6 + 1260\,x1^{2}\,x2\,x3^{2}\,x5\,x6 + 1260\,x1^{2}\,x2^{2}\,x4\,x5\,x6 + \\
&840\,x1^{3}\,x3\,x4\,x5\,x6 + 420\,x3^{3}\,x4^{2}\,x5\,x6 + 840\,x2\,x3\,x4^{3}\,x5\,x6 + 210\,x1\,x4^{4}\,x5\,x6 + 420\,x1^{3}\,x2\,x5^{2}\,x6 + \\
&105\,x3^{4}\,x5^{2}\,x6 + 1260\,x2\,x3^{2}\,x4\,x5^{2}\,x6 + 630\,x2^{2}\,x4^{2}\,x5^{2}\,x6 + 1260\,x1\,x3\,x4^{2}\,x5^{2}\,x6 + 420\,x2^{2}\,x3\,x5^{3}\,x6 + \\
&420\,x1\,x3^{2}\,x5^{3}\,x6 + 840\,x1\,x2\,x4\,x5^{3}\,x6 + 105\,x1^{2}\,x5^{4}\,x6 + 42\,x4\,x5^{5}\,x6 + 105\,x1\,x2^{4}\,x6^{2} + 630\,x1^{2}\,x2^{2}\,x3\,x6^{2} + \\
&210\,x1^{3}\,x3^{2}\,x6^{2} + 420\,x1^{3}\,x2\,x4\,x6^{2} + 105\,x3^{4}\,x4\,x6^{2} + 630\,x2\,x3^{2}\,x4^{2}\,x6^{2} + 210\,x2^{2}\,x4^{3}\,x6^{2} + \\
&420\,x1\,x3\,x4^{3}\,x6^{2} + 105\,x1^{4}\,x5\,x6^{2} + 420\,x2\,x3^{3}\,x5\,x6^{2} + 1260\,x2^{2}\,x3\,x4\,x5\,x6^{2} + 1260\,x1\,x3^{2}\,x4\,x5\,x6^{2} + \\
&1260\,x1\,x2\,x4^{2}\,x5\,x6^{2} + 210\,x2^{3}\,x5^{2}\,x6^{2} + 1260\,x1\,x2\,x3\,x5^{2}\,x6^{2} + 630\,x1^{2}\,x4\,x5^{2}\,x6^{2} + 210\,x4^{2}\,x5^{3}\,x6^{2} + \\
&105\,x3\,x5^{4}\,x6^{2} + 210\,x2^{2}\,x3^{2}\,x6^{3} + 140\,x1\,x3^{3}\,x6^{3} + 140\,x2^{3}\,x4\,x6^{3} + 840\,x1\,x2\,x3\,x4\,x6^{3} + 210\,x1^{2}\,x4^{2}\,x6^{3} + \\
&420\,x1\,x2^{2}\,x5\,x6^{3} + 420\,x1^{2}\,x3\,x5\,x6^{3} + 140\,x4^{3}\,x5\,x6^{3} + 420\,x3\,x4\,x5^{2}\,x6^{3} + 140\,x2\,x5^{3}\,x6^{3} + 105\,x1^{2}\,x2\,x6^{4} + \\
&105\,x3\,x4^{2}\,x6^{4} + 105\,x3^{2}\,x5\,x6^{4} + 210\,x2\,x4\,x5\,x6^{4} + 105\,x1\,x5^{2}\,x6^{4} + 42\,x2\,x3\,x6^{5} + 42\,x1\,x4\,x6^{5} + x6^{7} + \\
&42\,x1^{5}\,x2\,x7 + 105\,x2^{4}\,x3^{2}\,x7 + 420\,x1\,x2^{2}\,x3^{3}\,x7 + 105\,x1^{2}\,x3^{4}\,x7 + 42\,x2^{5}\,x4\,x7 + 840\,x1\,x2^{3}\,x3\,x4\,x7 + \\
&1260\,x1^{2}\,x2\,x3^{2}\,x4\,x7 + 630\,x1^{2}\,x2^{2}\,x4^{2}\,x7 + 420\,x1^{3}\,x3\,x4^{2}\,x7 + 140\,x3^{3}\,x4^{3}\,x7 + 210\,x2\,x3\,x4^{4}\,x7 + \\
&42\,x1\,x4^{5}\,x7 + 210\,x1\,x2^{4}\,x5\,x7 + 1260\,x1^{2}\,x2^{2}\,x3\,x5\,x7 + 420\,x1^{3}\,x3^{2}\,x5\,x7 + 840\,x1^{3}\,x2\,x4\,x5\,x7 + \\
&210\,x3^{4}\,x4\,x5\,x7 + 1260\,x2\,x3^{2}\,x4^{2}\,x5\,x7 + 420\,x2^{2}\,x4^{3}\,x5\,x7 + 840\,x1\,x3\,x4^{3}\,x5\,x7 + 105\,x1^{4}\,x5^{2}\,x7 + \\
&420\,x2\,x3^{3}\,x5^{2}\,x7 + 1260\,x2^{2}\,x3\,x4\,x5^{2}\,x7 + 1260\,x1\,x3^{2}\,x4\,x5^{2}\,x7 + 1260\,x1\,x2\,x4^{2}\,x5^{2}\,x7 + 140\,x2^{3}\,x5^{3}\,x7 + \\
&840\,x1\,x2\,x3\,x5^{3}\,x7 + 420\,x1^{2}\,x4\,x5^{3}\,x7 + 105\,x4^{2}\,x5^{4}\,x7 + 42\,x3\,x5^{5}\,x7 + 420\,x1^{2}\,x2^{3}\,x6\,x7 + \\
&840\,x1^{3}\,x2\,x3\,x6\,x7 + 42\,x3^{5}\,x6\,x7 + 210\,x1^{4}\,x4\,x6\,x7 + 840\,x2\,x3^{3}\,x4\,x6\,x7 + 1260\,x2^{2}\,x3\,x4^{2}\,x6\,x7 + \\
&1260\,x1\,x3^{2}\,x4^{2}\,x6\,x7 + 840\,x1\,x2\,x4^{3}\,x6\,x7 + 1260\,x2^{2}\,x3^{2}\,x5\,x6\,x7 + 840\,x1\,x3^{3}\,x5\,x6\,x7 + \\
&840\,x2^{3}\,x4\,x5\,x6\,x7 + 5040\,x1\,x2\,x3\,x4\,x5\,x6\,x7 + 1260\,x1^{2}\,x4^{2}\,x5\,x6\,x7 + 1260\,x1\,x2^{2}\,x5^{2}\,x6\,x7 + \\
&1260\,x1^{2}\,x3\,x5^{2}\,x6\,x7 + 420\,x4^{3}\,x5^{2}\,x6\,x7 + 840\,x3\,x4\,x5^{3}\,x6\,x7 + 210\,x2\,x5^{4}\,x6\,x7 + 420\,x2^{3}\,x3\,x6^{2}\,x7 + \\
&1260\,x1\,x2\,x3^{2}\,x6^{2}\,x7 + 1260\,x1\,x2^{2}\,x4\,x6^{2}\,x7 + 1260\,x1^{2}\,x3\,x4\,x6^{2}\,x7 + 105\,x4^{4}\,x6^{2}\,x7 + \\
&1260\,x1^{2}\,x2\,x5\,x6^{2}\,x7 + 1260\,x3\,x4^{2}\,x5\,x6^{2}\,x7 + 630\,x3^{2}\,x5^{2}\,x6^{2}\,x7 + 1260\,x2\,x4\,x5^{2}\,x6^{2}\,x7 + \\
&420\,x1\,x5^{3}\,x6^{2}\,x7 + 140\,x1^{3}\,x6^{3}\,x7 + 420\,x3^{2}\,x4\,x6^{3}\,x7 + 420\,x2\,x4^{2}\,x6^{3}\,x7 + 840\,x2\,x3\,x5\,x6^{3}\,x7 + \\
&840\,x1\,x4\,x5\,x6^{3}\,x7 + 105\,x2^{2}\,x6^{4}\,x7 + 210\,x1\,x3\,x6^{4}\,x7 + 42\,x5\,x6^{5}\,x7 + 210\,x1^{3}\,x2^{2}\,x7^{2} + 105\,x1^{4}\,x3\,x7^{2} + \\
&105\,x2\,x3^{4}\,x7^{2} + 630\,x2^{2}\,x3^{2}\,x4\,x7^{2} + 420\,x1\,x3^{3}\,x4\,x7^{2} + 210\,x2^{3}\,x4^{2}\,x7^{2} + 1260\,x1\,x2\,x3\,x4^{2}\,x7^{2} + \\
&210\,x1^{2}\,x4^{3}\,x7^{2} + 420\,x2^{3}\,x3\,x5\,x7^{2} + 1260\,x1\,x2\,x3^{2}\,x5\,x7^{2} + 1260\,x1\,x2^{2}\,x4\,x5\,x7^{2} + 1260\,x1^{2}\,x3\,x4\,x5\,x7^{2} + \\
&105\,x4^{4}\,x5\,x7^{2} + 630\,x1^{2}\,x2\,x5^{2}\,x7^{2} + 630\,x3\,x4^{2}\,x5^{2}\,x7^{2} + 210\,x3^{2}\,x5^{3}\,x7^{2} + 420\,x2\,x4\,x5^{3}\,x7^{2} + \\
&105\,x1\,x5^{4}\,x7^{2} + 105\,x2^{4}\,x6\,x7^{2} + 1260\,x1\,x2^{2}\,x3\,x6\,x7^{2} + 630\,x1^{2}\,x3^{2}\,x6\,x7^{2} + 1260\,x1^{2}\,x2\,x4\,x6\,x7^{2} + \\
&420\,x3\,x4^{3}\,x6\,x7^{2} + 420\,x1^{3}\,x5\,x6\,x7^{2} + 1260\,x3^{2}\,x4\,x5\,x6\,x7^{2} + 1260\,x2\,x4^{2}\,x5\,x6\,x7^{2} + 1260\,x2\,x3\,x5^{2}\,x6\,x7^{2} + \\
&1260\,x1\,x4\,x5^{2}\,x6\,x7^{2} + 210\,x3^{3}\,x6^{2}\,x7^{2} + 1260\,x2\,x3\,x4\,x6^{2}\,x7^{2} + 630\,x1\,x4^{2}\,x6^{2}\,x7^{2} + 630\,x2^{2}\,x5\,x6^{2}\,x7^{2} + \\
&1260\,x1\,x3\,x5\,x6^{2}\,x7^{2} + 420\,x1\,x2\,x6^{3}\,x7^{2} + 210\,x5^{2}\,x6^{3}\,x7^{2} + 105\,x4\,x6^{4}\,x7^{2} + 140\,x1\,x2^{3}\,x7^{3} + \\
&420\,x1^{2}\,x2\,x3\,x7^{3} + 140\,x1^{3}\,x4\,x7^{3} + 210\,x3^{2}\,x4^{2}\,x7^{3} + 140\,x2\,x4^{3}\,x7^{3} + 140\,x3^{3}\,x5\,x7^{3} + 840\,x2\,x3\,x4\,x5\,x7^{3} + \\
&420\,x1\,x4^{2}\,x5\,x7^{3} + 210\,x2^{2}\,x5^{2}\,x7^{3} + 420\,x1\,x3\,x5^{2}\,x7^{3} + 420\,x2\,x3^{2}\,x6\,x7^{3} + 420\,x2^{2}\,x4\,x6\,x7^{3} + \\
&840\,x1\,x3\,x4\,x6\,x7^{3} + 840\,x1\,x2\,x5\,x6\,x7^{3} + 140\,x5^{3}\,x6\,x7^{3} + 210\,x1^{2}\,x6^{2}\,x7^{3} + 420\,x4\,x5\,x6^{2}\,x7^{3} + \\
&140\,x3\,x6^{3}\,x7^{3} + 105\,x2^{2}\,x3\,x7^{4} + 105\,x1\,x3^{2}\,x7^{4} + 210\,x1\,x2\,x4\,x7^{4} + 105\,x1^{2}\,x5\,x7^{4} + 105\,x4\,x5^{2}\,x7^{4} + \\
&105\,x4^{2}\,x6\,x7^{4} + 210\,x3\,x5\,x6\,x7^{4} + 105\,x2\,x6^{2}\,x7^{4} + 42\,x3\,x4\,x7^{5} + 42\,x2\,x5\,x7^{5} + 42\,x1\,x6\,x7^{5} + x7^{7}
\end{aligned}
$$

この後は F_{42} の固定体が Q (ζ) であることを使うので,
x1 ~ x7 に 原始元v による表現 `$x1` ~ `$x7` を代入して,
h (x) の式を求めます. (L_0 のみ出力します)

```
rule = {x1 → $x1, x2 → $x2, x3 → $x3, x4 → $x4, x5 → $x5, x6 → $x6, x7 → $x7};
l0 = l0 /. rule // Expand // PolynomialMod[#, V[v]] &
l1 = l1 /. rule // Expand // PolynomialMod[#, V[v]] &;
l2 = l2 /. rule // Expand // PolynomialMod[#, V[v]] &;
l3 = l3 /. rule // Expand // PolynomialMod[#, V[v]] &;
l4 = l4 /. rule // Expand // PolynomialMod[#, V[v]] &;
l5 = l5 /. rule // Expand // PolynomialMod[#, V[v]] &;
l6 = l6 /. rule // Expand // PolynomialMod[#, V[v]] &;
```

Out[∘]=

-49 735

l_0は確かに有理数です. それでは *h(x)* を「単純計算」で求めましょう.

In[∘]:=

```
h[x_] =
  (x - l1) (x - l2) (x - l3) (x - l4) (x - l5) (x - l6) // Expand // PolynomialMod[#, V[v]] &
IrreduciblePolynomialQ[%]
```

Out[∘]=

$$11\,555\,998\,612\,268\,398\,934\,771\,395\,165 - 272\,298\,401\,423\,248\,581\,462\,977\,x + 5\,440\,900\,067\,981\,861\,098\,x^2 - 108\,827\,153\,500\,758\,x^3 + 2\,187\,145\,331\,x^4 - 49\,735\,x^5 + x^6$$

Out[∘]=

True

確かに有理数係数で既約な6次式です. 因みに

「$l_0+l_1+l_2+l_3+l_4+l_5+l_6= (x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7)^7 =0$」

が成り立つので, x^5 の係数はl_0と等しいです. また Magma を使って求めると Galois群はC_6です.

§2. $h(x)$ を解いて R_1~R_6の候補を求める

2-1. h(x)を解く

(#1) よりσで $l_1 \sim l_6$は動かず, また τにより「$l_1 \to l_3 \to l_2 \to l_6 \to l_4 \to l_5 \to l_1$」と動くので, F_{21}=<σ, τ^2> によって l_1+l_2+l_4, l_3+l_5+l_6, l_3l_5+l_5l_6+l_6l_3, $l_1l_2l_4$,$l_3l_5l_6$ は動きません. 故にこれらは F_{21}の固定体 Mに入ります. Mを求めるために判別式を考えると,

In[ ]:=
```
Discriminant[h[x], x] // Sqrt
```

Out[ ]=

$$188\,572\,246\,077\,133\,698\,414\,704\,745\,872\,532\,190\,638\,223\,478\,708\,796\,741\,136\,210\,046\,635\,857\,125\, i\, \sqrt{91}$$

よってh (x) はQ $\left(\sqrt{91}\ i\right)$ で因数分解できます.

In[ ]:=
```
hx = Factor[h[x], Extension → Sqrt[-91]]
```

Out[ ]=

$$\frac{1}{4}\left(-214\,994\,868\,180\,323\, i + 114\,886\,719\,129\sqrt{91} + \left(2\,502\,226\,160\, i - 634\,262\,566\sqrt{91}\right) x + \left(49\,735\, i + 13\,377\sqrt{91}\right) x^2 - 2\, i\, x^3\right) \left(214\,994\,868\,180\,323\, i + 114\,886\,719\,129\sqrt{91} + \left(-2\,502\,226\,160\, i - 634\,262\,566\sqrt{91}\right) x + \left(-49\,735\, i + 13\,377\sqrt{91}\right) x^2 + 2\, i\, x^3\right)$$

h(x) = h1*h2 (h1, h2 は3次式) と分けると

In[ ]:=
```
h1 = - (List @@ hx) [[2]]
h2 = (List @@ hx) [[3]]
```

Out[ ]=

$$214\,994\,868\,180\,323\, i - 114\,886\,719\,129\sqrt{91} - \left(2\,502\,226\,160\, i - 634\,262\,566\sqrt{91}\right) x - \left(49\,735\, i + 13\,377\sqrt{91}\right) x^2 + 2\, i\, x^3$$

Out[ ]=

$$214\,994\,868\,180\,323\, i + 114\,886\,719\,129\sqrt{91} + \left(-2\,502\,226\,160\, i - 634\,262\,566\sqrt{91}\right) x + \left(-49\,735\, i + 13\,377\sqrt{91}\right) x^2 + 2\, i\, x^3$$

h1 = 0 の解を {x11, x12,x13}, h2 = 0 の解を {x21,x22,x23} とおき, {l1,l2,l3,l4,l5,l6}={ x11, x12, x21, x13, x22, x23} と置きます. (後で使うためにl1~l6を$l1~$l6とします. また$l1のみを表示しています)

In[ ]:=
```
{$l1, $l2, $l3, $l4, $l5, $l6} =
  {x /. Solve[h1 == 0, x, Cubics → True], x /. Solve[h2 == 0, x, Cubics → True]} //
   Flatten[#, 1][[{1, 2, 4, 3, 5, 6}]] &; $l1
```

Out[ ]=

$$-\frac{343}{6} i \left(145\, i + 39\sqrt{91}\right) - \frac{1}{3\times 2^{2/3}} 49\, i \left(24\,851\,368\,048\, i + 403\,803\,414\sqrt{91} + 3\sqrt{6\left(-11\,834\,381\,343\,207\,751\,614 - 86\,180\,080\,543\,887\,512\, i\sqrt{91}\right)}\right)^{1/3} + \left(343\, i \left(35\,789 + 73\,629\, i\sqrt{91}\right)\right) \Big/ \left(3\left(2\left(24\,851\,368\,048\, i + 403\,803\,414\sqrt{91} + 3\sqrt{6\left(-11\,834\,381\,343\,207\,751\,614 - 86\,180\,080\,543\,887\,512\, i\sqrt{91}\right)}\right)\right)^{1/3}\right)$$

2-2. R_1~R_6とr1~r6の値を求める

もちろん，ここで，$\$l_1$~$\$l_6$ が本来あるべき l_1~l_6 と一致しているとは限りません.
しかし $\{\$l_1, \$l_2, \$l_4\}$ と $\{\$l_3, \$l_5, \$l_6\}$ は同じ3次方程式の解であることから,
正しい $(\$l_1, \$l_2, \$l_3, \$l_4, \$l_5, \$l_6)$ は，$\$l_i = i$ と省略すると,
(1, 2, 3, 4, 5, 6), (1, 2, 3, 4, 6, 5), (1, 2, 5, 4, 3, 6),
(1, 2, 5, 4, 6, 3), ..., (3, 5, 1, 6, 2, 4), ... (6, 5, 4, 3, 2, 1)
などの 3！×3！×2！の72通りのいずれか.

ここで例えば(1,2,3,4,5,6)をτで移した
(3,6,2,5,1,4),(2,4,6,1,3,5),(6,5,4,3,2,1),(4,1,5,2,6,3),(5,3,1,6,4,2)

は同じ R_1~R_6の集合を与えます．故に本質的に異なる $(\$l_1, \$l_2, \$l_3, \$l_4, \$l_5, \$l_6)$ は72通り/6通り=12通り　となります．これらに対し R_1~R_6を求め，さらに束縛条件$\{r_1 r_2^2+r_4 r_3^2, r_3 r_1^2+r_2 r_4^2\}=\{\frac{7}{2}(1-7\sqrt{13}), \frac{7}{2}(1+7\sqrt{13})\}$ (F42_Solutions.nb 補足参照)を満たす r1~r6を求めることも可能ですが，かなり大変なので，F42_Solutions.nb と同様に，数値計算を使って求めたいと思います．まずは　前節の$l1~$l6の近似値を求めます.

In[ ]:=
```
N[{$l1, $l2, $l3, $l4, $l5, $l6}]
```

Out[ ]=
```
{-30 207.8 - 37 806.6 i, 11 916.3 - 45 404.7 i, 43 159. - 19 407.1 i,
 43 159. + 19 407.1 i, -30 207.8 + 37 806.6 i, 11 916.3 + 45 404.7 i}
```

l1~l6はvの式で表されているので，vの値を代入して，近似値を求めます.

In[ ]:=
```
vs = v /. NSolve[V[v] == 0, v];
{l1, l2, l3, l4, l5, l6} /. {v → vs[[1]]} // N
```

Out[ ]=
```
{11 916.2 + 45 404.7 i, 43 159. - 19 407.1 i, -30 207.7 - 37 806.6 i,
 -30 207.8 + 37 806.6 i, 43 158.9 + 19 407.1 i, 11 916.3 - 45 404.7 i}
```

直前の2つの結果から「(l1, l2, l3, l4, l5, l6) = ($l6, $l3, $l1, $l5, $l4, $l2) 」となります．したがって(#2)より，R1~R6 の値は次のようになります．(R1のみ表示しています)

In[ ]:=
```
ξ = Cos[2 Pi / 7] + I Sin[2 Pi / 7];
l1 = $l6; l2 = $l3; l3 = $l1; l4 = $l5; l5 = $l4; l6 = $l2;
R1 = l0 + l1 ξ + l2 ξ^2 + l3 ξ^3 + l4 ξ^4 + l5 ξ^5 + l6 ξ^6 // Simplify
R2 = l0 + l4 ξ + l1 ξ^2 + l5 ξ^3 + l2 ξ^4 + l6 ξ^5 + l3 ξ^6 // Simplify;
R3 = l0 + l5 ξ + l3 ξ^2 + l1 ξ^3 + l6 ξ^4 + l4 ξ^5 + l2 ξ^6 // Simplify;
R4 = l0 + l2 ξ + l4 ξ^2 + l6 ξ^3 + l1 ξ^4 + l3 ξ^5 + l5 ξ^6 // Simplify;
R5 = l0 + l3 ξ + l6 ξ^2 + l2 ξ^3 + l5 ξ^4 + l1 ξ^5 + l4 ξ^6 // Simplify;
R6 = l0 + l6 ξ + l5 ξ^2 + l4 ξ^3 + l3 ξ^4 + l2 ξ^5 + l1 ξ^6 // Simplify;
```

Out[ ]=

$$\frac{49}{12}\left(-12\,180 + \right.$$
$$i\left(-2030\, i + 546\sqrt{91} - \left(7\left(-i+\sqrt{3}\right)\left(35\,789\, i + 73\,629\sqrt{91}\right)\right)\right) \Big/ \left(6\,212\,842\,012\, i - \frac{201\,901\,707\sqrt{91}}{2} + \right.$$

$$\frac{3}{2}\sqrt{39\left(-455168513200298139+3314618482457212\, i\sqrt{91}\right)}\Bigg)^{1/3}+$$

$$2^{2/3}\left(1-i\sqrt{3}\right)\Bigg(12425684024\, i-201901707\sqrt{91}+$$

$$3\sqrt{39\left(-455168513200298139+3314618482457212\, i\sqrt{91}\right)}\Bigg)^{1/3}\Bigg)$$

$$\left(i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)+2\,i\,\Bigg(7\left(-145\,i+39\sqrt{91}\right)+\left(7\left(35789-73629\,i\sqrt{91}\right)\right)\Big/$$

$$\Bigg(6212842012\,i-\frac{201901707\sqrt{91}}{2}+$$

$$\frac{3}{2}\sqrt{39\left(-455168513200298139+3314618482457212\, i\sqrt{91}\right)}\Bigg)^{1/3}-2^{2/3}\Bigg(12425684024\,i-$$

$$201901707\sqrt{91}+3\sqrt{39\left(-455168513200298139+3314618482457212\, i\sqrt{91}\right)}\Bigg)^{1/3}\Bigg)$$

$$\left(i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^2-2\,i\,\Bigg(7\left(145\,i+39\sqrt{91}\right)-\left(7\left(35789+73629\,i\sqrt{91}\right)\right)\Big/$$

$$\Bigg(6212842012\,i+\frac{201901707\sqrt{91}}{2}+$$

$$\frac{3}{2}\sqrt{39\left(-455168513200298139-3314618482457212\, i\sqrt{91}\right)}\Bigg)^{1/3}+2^{2/3}\Bigg(12425684024\,i+$$

$$201901707\sqrt{91}+3\sqrt{39\left(-455168513200298139-3314618482457212\, i\sqrt{91}\right)}\Bigg)^{1/3}\Bigg)$$

$$\left(i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^3+i\,\Bigg(-2030\,i+546\sqrt{91}+\left(7\left(i+\sqrt{3}\right)\left(35789\,i+73629\sqrt{91}\right)\right)\Big/$$

$$\Bigg(6212842012\,i-\frac{201901707\sqrt{91}}{2}+$$

$$\frac{3}{2}\sqrt{39\left(-455168513200298139+3314618482457212\, i\sqrt{91}\right)}\Bigg)^{1/3}+$$

$$2^{2/3}\left(1+i\sqrt{3}\right)\Bigg(12425684024\, i-201901707\sqrt{91}+$$

$$3\sqrt{39\left(-455168513200298139+3314618482457212\, i\sqrt{91}\right)}\Bigg)^{1/3}\Bigg)$$

$$\left(i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^4-i\,\Bigg(2030\,i+546\sqrt{91}+\left(7\left(1+i\sqrt{3}\right)\left(35789+73629\,i\sqrt{91}\right)\right)\Big/$$

$$\Bigg(6212842012\,i+\frac{201901707\sqrt{91}}{2}+$$

$$\frac{3}{2}\sqrt{39\left(-455168513200298139-3314618482457212\, i\sqrt{91}\right)}\Bigg)^{1/3}+$$

$$i\,2^{2/3}\left(i+\sqrt{3}\right)\Bigg(12425684024\, i+201901707\sqrt{91}+$$

$$3\,\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,\mathbb{i}\,\sqrt{91}\right)}\,\Big)^{1/3}\Big)$$

$$\left(\mathbb{i}\,\mathrm{Cos}\left[\frac{3\,\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\,\pi}{14}\right]\right)^{5}-\mathbb{i}\,\Big(2030\,\mathbb{i}+546\,\sqrt{91}+\left(7\left(\mathbb{i}+\sqrt{3}\right)\left(-35\,789\,\mathbb{i}+73\,629\,\sqrt{91}\right)\right)\Big/$$

$$\Big(6\,212\,842\,012\,\mathbb{i}+\frac{201\,901\,707\,\sqrt{91}}{2}+$$

$$\frac{3}{2}\,\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,\mathbb{i}\,\sqrt{91}\right)}\,\Big)^{1/3}-$$

$$2^{2/3}\left(1+\mathbb{i}\,\sqrt{3}\right)\Big(12\,425\,684\,024\,\mathbb{i}+201\,901\,707\,\sqrt{91}+$$

$$3\,\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,\mathbb{i}\,\sqrt{91}\right)}\,\Big)^{1/3}\Big)\Big(\mathbb{i}\,\mathrm{Cos}\Big[$$

$$\frac{3\,\pi}{14}\Big]+\mathrm{Sin}\left[\frac{3\,\pi}{14}\right]\Big)^{6}\Big)$$

次にR1~R6の近似値を求めます.

In[∘]:=
```
N[{R1, R2, R3, R4, R5, R6}]
```

Out[∘]=

$$\{-0.0397559-5.94203\times10^{-11}\,\mathbb{i},\ -334\,967.+9.28443\times10^{-12}\,\mathbb{i},\ 25.2072+3.71377\times10^{-11}\,\mathbb{i},\ -7915.77+2.59964\times10^{-11}\,\mathbb{i},\ -5986.19+0.\,\mathbb{i},\ 698.588-3.34239\times10^{-11}\,\mathbb{i}\}$$

どうやら, R1～R6は全て実数としても良いようです. そして (D5_Solutions.nb) と同様にして

「$R_1 \sim R_6$ が実数のときは $r_1 \sim r_6$ が全て実数としても一般性を失わない」から,
r1～r6の値は次のようになります.

In[∘]:=
```
real7thRoot[x_] := If[Re[N[x]] > 0, x^(1/7), -(-x)^(1/7)];
{r1, r2, r3, r4, r5, r6} = real7thRoot /@ {R1, R2, R3, R4, R5, R6};
```

2-3. r_1~r_6を代入して，方程式の解 x_1~ x_7 を求める

r1~r6の定義式の逆変換を求めて，x1~x7をr1~r6の式で表すことができます．それに代入します．(x1のみ表示してます)

In[∘]:=
```
x1 = 1 / 7 (r1 + r2 + r3 + r4 + r5 + r6)
x2 = 1 / 7 (ξ^6 r1 + ξ^5 r2 + ξ^4 r3 + ξ^3 r4 + ξ^2 r5 + ξ r6);
x3 = 1 / 7 (ξ^5 r1 + ξ^3 r2 + ξ r3 + ξ^6 r4 + ξ^4 r5 + ξ^2 r6);
x4 = 1 / 7 (ξ^4 r1 + ξ r2 + ξ ^5 r3 + ξ^2 r4 + ξ^6 r5 + ξ^3 r6);
x5 = 1 / 7 (ξ^3 r1 + ξ^6 r2 + ξ ^2 r3 + ξ^5 r4 + ξ r5 + ξ^4 r6);
x6 = 1 / 7 (ξ^2 r1 + ξ^4 r2 + ξ ^6 r3 + ξ r4 + ξ^3 r5 + ξ^5 r6);
x7 = 1 / 7 (ξ r1 + ξ^2 r2 + ξ ^3 r3 + ξ^4 r4 + ξ^5 r5 + ξ^6 r6);
```

Out[∘]=

$$\frac{1}{7}\left(-\frac{1}{2^{2/7}}\times 7^{2/7}\left(\frac{1}{3}\left(12180-\right.\right.\right.$$

$$i\left(-2030\,i+546\sqrt{91}+\frac{7\left(i+\sqrt{3}\right)\left(35789\,i+73629\sqrt{91}\right)}{\left(6212842012\,i-\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139+3314618482457212\,i\sqrt{91}\right)}\right)^{1/3}}+\right.$$

$$2^{2/3}\left(1+i\sqrt{3}\right)\left(12425684024\,i-201901707\sqrt{91}+\right.$$

$$\left.\left.3\sqrt{39\left(-455168513200298139+3314618482457212\,i\sqrt{91}\right)}\right)^{1/3}\right)\left(i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)-$$

$$i\left(-2030\,i+546\sqrt{91}-\frac{7\left(-i+\sqrt{3}\right)\left(35789\,i+73629\sqrt{91}\right)}{\left(6212842012\,i-\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139+3314618482457212\,i\sqrt{91}\right)}\right)^{1/3}}+\right.$$

$$2^{2/3}\left(1-i\sqrt{3}\right)\left(12425684024\,i-201901707\sqrt{91}+\right.$$

$$\left.\left.3\sqrt{39\left(-455168513200298139+3314618482457212\,i\sqrt{91}\right)}\right)^{1/3}\right)\left(i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^{2}+$$

$$i\left(2030\,i+546\sqrt{91}+\frac{7\left(1+i\sqrt{3}\right)\left(35789+73629\,i\sqrt{91}\right)}{\left(6212842012\,i+\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139-3314618482457212\,i\sqrt{91}\right)}\right)^{1/3}}+\right.$$

$$\left.i\,2^{2/3}\left(i+\sqrt{3}\right)\left(12425684024\,i+201901707\sqrt{91}+3\sqrt{39\left(-455168513200298139-3314618482457212\,i\sqrt{91}\right)}\right)^{1/3}\right)$$

$$\left(i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^{3}-2\,i\left(7\left(-145\,i+39\sqrt{91}\right)+\right.$$

$$\frac{7\left(35789-73629\,i\sqrt{91}\right)}{\left(6212842012\,i-\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139+3314618482457212\,i\sqrt{91}\right)}\right)^{1/3}}-2^{2/3}$$

$$\left.\left(12425684024\,i-201901707\sqrt{91}+3\sqrt{39\left(-455168513200298139+3314618482457212\,i\sqrt{91}\right)}\right)^{1/3}\right)\left(i\right.$$

$$\left.\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^{4}+$$

$$\mathrm{i}\ \Bigg(2030\,\mathrm{i}+546\sqrt{91}+\frac{7\left(\mathrm{i}+\sqrt{3}\right)\left(-35\,789\,\mathrm{i}+73\,629\sqrt{91}\right)}{\left(6\,212\,842\,012\,\mathrm{i}+\frac{201\,901\,707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}}-$$

$$2^{2/3}\left(1+\mathrm{i}\sqrt{3}\right)\left(12\,425\,684\,024\,\mathrm{i}+201\,901\,707\sqrt{91}+3\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}\Bigg)$$

$$\left(\mathrm{i}\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^{5}+2\,\mathrm{i}\ \Bigg(7\left(145\,\mathrm{i}+39\sqrt{91}\right)-$$

$$\frac{7\left(35\,789+73\,629\,\mathrm{i}\sqrt{91}\right)}{\left(6\,212\,842\,012\,\mathrm{i}+\frac{201\,901\,707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}}+2^{2/3}$$

$$\left(12\,425\,684\,024\,\mathrm{i}+201\,901\,707\sqrt{91}+3\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}\Bigg)$$

$$\left(\mathrm{i}\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^{6}\Bigg)\Bigg)^{1/7}-\frac{1}{2^{2/7}}\times 7^{2/7}\ \Bigg(\frac{1}{3}\ \Bigg(12\,180-\mathrm{i}\ \Bigg(-2030\,\mathrm{i}+546\sqrt{91}-$$

$$\frac{7\left(-\mathrm{i}+\sqrt{3}\right)\left(35\,789\,\mathrm{i}+73\,629\sqrt{91}\right)}{\left(6\,212\,842\,012\,\mathrm{i}-\frac{201\,901\,707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}}+$$

$$2^{2/3}\left(1-\mathrm{i}\sqrt{3}\right)\left(12\,425\,684\,024\,\mathrm{i}-201\,901\,707\sqrt{91}+3\sqrt{39\left(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}\Bigg)$$

$$\left(\mathrm{i}\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)-2\,\mathrm{i}\ \Bigg(7\left(-145\,\mathrm{i}+39\sqrt{91}\right)+$$

$$\frac{7\left(35\,789-73\,629\,\mathrm{i}\sqrt{91}\right)}{\left(6\,212\,842\,012\,\mathrm{i}-\frac{201\,901\,707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}}-$$

$$2^{2/3}\left(12\,425\,684\,024\,\mathrm{i}-201\,901\,707\sqrt{91}+3\sqrt{39\left(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}\Bigg)$$

$$\left(\mathrm{i}\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^{2}+2\,\mathrm{i}\ \Bigg(7\left(145\,\mathrm{i}+39\sqrt{91}\right)-$$

$$\frac{7\left(35\,789+73\,629\,\mathrm{i}\sqrt{91}\right)}{\left(6\,212\,842\,012\,\mathrm{i}+\frac{201\,901\,707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}}+$$

$$2^{2/3}\left(12\,425\,684\,024\,\mathrm{i}+201\,901\,707\sqrt{91}+3\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}\Bigg)$$

$$\left(\mathrm{i}\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^{3}-\mathrm{i}\ \Bigg(-2030\,\mathrm{i}+546\sqrt{91}+$$

$$\frac{7\left(\mathrm{i}+\sqrt{3}\right)\left(35\,789\,\mathrm{i}+73\,629\sqrt{91}\right)}{\left(6\,212\,842\,012\,\mathrm{i}-\frac{201\,901\,707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}}+$$

$$2^{2/3}\left(1+i\sqrt{3}\right)\left(12\,425\,684\,024\,i-201\,901\,707\sqrt{91}+\right.$$

$$\left.3\sqrt{39\left(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,i\sqrt{91}\right)}\right)^{1/3}\Bigg)\left(i\,\text{Cos}\left[\frac{3\pi}{14}\right]+\text{Sin}\left[\frac{3\pi}{14}\right]\right)^{4}+$$

$$i\left(2030\,i+546\sqrt{91}+\frac{7\left(1+i\sqrt{3}\right)\left(35\,789+73\,629\,i\sqrt{91}\right)}{\left(6\,212\,842\,012\,i+\frac{201\,901\,707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,i\sqrt{91}\right)}\right)^{1/3}}+\right.$$

$$i\,2^{2/3}\left(i+\sqrt{3}\right)\left(12\,425\,684\,024\,i+201\,901\,707\sqrt{91}+\right.$$

$$\left.3\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,i\sqrt{91}\right)}\right)^{1/3}\Bigg)\left(i\,\text{Cos}\left[\frac{3\pi}{14}\right]+\text{Sin}\left[\frac{3\pi}{14}\right]\right)^{5}+$$

$$i\left(2030\,i+546\sqrt{91}+\frac{7\left(i+\sqrt{3}\right)\left(-35\,789\,i+73\,629\sqrt{91}\right)}{\left(6\,212\,842\,012\,i+\frac{201\,901\,707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,i\sqrt{91}\right)}\right)^{1/3}}-\right.$$

$$2^{2/3}\left(1+i\sqrt{3}\right)\left(12\,425\,684\,024\,i+201\,901\,707\sqrt{91}+3\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,i\sqrt{91}\right)}\right)^{1/3}\Bigg)$$

$$\left(i\,\text{Cos}\left[\frac{3\pi}{14}\right]+\text{Sin}\left[\frac{3\pi}{14}\right]\right)^{6}\Bigg)\Bigg)^{1/7}-\frac{1}{2^{2/7}}\times 7^{2/7}\left(\frac{1}{3}\left(12\,180-2\,i\left(7\left(-145\,i+39\sqrt{91}\right)+\right.\right.\right.$$

$$\frac{7\left(35\,789-73\,629\,i\sqrt{91}\right)}{\left(6\,212\,842\,012\,i-\frac{201\,901\,707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,i\sqrt{91}\right)}\right)^{1/3}}-$$

$$2^{2/3}\left(12\,425\,684\,024\,i-201\,901\,707\sqrt{91}+3\sqrt{39\left(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,i\sqrt{91}\right)}\right)^{1/3}\Bigg)\Bigg(i$$

$$\text{Cos}\left[\frac{3\pi}{14}\right]+\text{Sin}\left[\frac{3\pi}{14}\right]\Bigg)-$$

$$i\left(-2030\,i+546\sqrt{91}+\frac{7\left(i+\sqrt{3}\right)\left(35\,789\,i+73\,629\sqrt{91}\right)}{\left(6\,212\,842\,012\,i-\frac{201\,901\,707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,i\sqrt{91}\right)}\right)^{1/3}}+\right.$$

$$2^{2/3}\left(1+i\sqrt{3}\right)\left(12\,425\,684\,024\,i-201\,901\,707\sqrt{91}+\right.$$

$$\left.3\sqrt{39\left(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,i\sqrt{91}\right)}\right)^{1/3}\Bigg)\left(i\,\text{Cos}\left[\frac{3\pi}{14}\right]+\text{Sin}\left[\frac{3\pi}{14}\right]\right)^{2}+$$

$$i\left(2030\,i+546\sqrt{91}+\frac{7\left(i+\sqrt{3}\right)\left(-35\,789\,i+73\,629\sqrt{91}\right)}{\left(6\,212\,842\,012\,i+\frac{201\,901\,707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,i\sqrt{91}\right)}\right)^{1/3}}-\right.$$

$$2^{2/3}\left(1+i\sqrt{3}\right)\left(12\,425\,684\,024\,i+201\,901\,707\sqrt{91}+\right.$$

$$\left.3\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,i\sqrt{91}\right)}\right)^{1/3}\Bigg)\left(i\,\text{Cos}\left[\frac{3\pi}{14}\right]+\text{Sin}\left[\frac{3\pi}{14}\right]\right)^{3}-$$

$$i\left(-2030\,i+546\sqrt{91}-\frac{7\left(-i+\sqrt{3}\right)\left(35\,789\,i+73\,629\sqrt{91}\right)}{\left(6\,212\,842\,012\,i-\frac{201\,901\,707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,i\sqrt{91}\right)}\right)^{1/3}}+\right.$$

$$2^{2/3}\ (1-\mathrm{i}\ \sqrt{3})\ \left(12\,425\,684\,024\ \mathrm{i}-201\,901\,707\ \sqrt{91}+3\ \sqrt{39\ (-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\ \mathrm{i}\ \sqrt{91})}\right)^{1/3}\Bigg)$$

$$\left(\mathrm{i}\ \mathrm{Cos}\left[\frac{3\,\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\,\pi}{14}\right]\right)^{4}+2\ \mathrm{i}\ \Bigg(7\ (145\ \mathrm{i}+39\ \sqrt{91})-$$

$$\frac{7\ (35\,789+73\,629\ \mathrm{i}\ \sqrt{91})}{\left(6\,212\,842\,012\ \mathrm{i}+\frac{201\,901\,707\ \sqrt{91}}{2}+\frac{3}{2}\ \sqrt{39\ (-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\ \mathrm{i}\ \sqrt{91})}\right)^{1/3}}+$$

$$2^{2/3}\ \left(12\,425\,684\,024\ \mathrm{i}+201\,901\,707\ \sqrt{91}+3\ \sqrt{39\ (-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\ \mathrm{i}\ \sqrt{91})}\right)^{1/3}\Bigg)\ \Bigg(\mathrm{i}$$

$$\mathrm{Cos}\left[\frac{3\,\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\,\pi}{14}\right]\Bigg)^{5}+$$

$$\mathrm{i}\ \Bigg(2030\ \mathrm{i}+546\ \sqrt{91}+\frac{7\ (1+\mathrm{i}\ \sqrt{3})\ (35\,789+73\,629\ \mathrm{i}\ \sqrt{91})}{\left(6\,212\,842\,012\ \mathrm{i}+\frac{201\,901\,707\ \sqrt{91}}{2}+\frac{3}{2}\ \sqrt{39\ (-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\ \mathrm{i}\ \sqrt{91})}\right)^{1/3}}+$$

$$\mathrm{i}\ 2^{2/3}\ (\mathrm{i}+\sqrt{3})\ \left(12\,425\,684\,024\ \mathrm{i}+201\,901\,707\ \sqrt{91}+3\ \sqrt{39\ (-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\ \mathrm{i}\ \sqrt{91})}\right)^{1/3}\Bigg)$$

$$\left(\mathrm{i}\ \mathrm{Cos}\left[\frac{3\,\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\,\pi}{14}\right]\right)^{6}\Bigg)\Bigg)^{1/7}+\left(\frac{7}{2}\right)^{2/7}\ \Bigg(\frac{1}{3}\ \Bigg(-12\,180-$$

$$\mathrm{i}\ \Bigg(2030\ \mathrm{i}+546\ \sqrt{91}+\frac{7\ (1+\mathrm{i}\ \sqrt{3})\ (35\,789+73\,629\ \mathrm{i}\ \sqrt{91})}{\left(6\,212\,842\,012\ \mathrm{i}+\frac{201\,901\,707\ \sqrt{91}}{2}+\frac{3}{2}\ \sqrt{39\ (-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\ \mathrm{i}\ \sqrt{91})}\right)^{1/3}}+$$

$$\mathrm{i}\ 2^{2/3}\ (\mathrm{i}+\sqrt{3})\ \left(12\,425\,684\,024\ \mathrm{i}+201\,901\,707\ \sqrt{91}+3\ \sqrt{39\ (-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\ \mathrm{i}\ \sqrt{91})}\right)^{1/3}\Bigg)$$

$$\left(\mathrm{i}\ \mathrm{Cos}\left[\frac{3\,\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\,\pi}{14}\right]\right)-2\ \mathrm{i}\ \Bigg(7\ (145\ \mathrm{i}+39\ \sqrt{91})-$$

$$\frac{7\ (35\,789+73\,629\ \mathrm{i}\ \sqrt{91})}{\left(6\,212\,842\,012\ \mathrm{i}+\frac{201\,901\,707\ \sqrt{91}}{2}+\frac{3}{2}\ \sqrt{39\ (-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\ \mathrm{i}\ \sqrt{91})}\right)^{1/3}}+2^{2/3}\ \Bigg(12\,425\,684\,024\ \mathrm{i}+$$

$$201\,901\,707\ \sqrt{91}+3\ \sqrt{39\ (-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\ \mathrm{i}\ \sqrt{91})}\Bigg)^{1/3}\Bigg)\ \left(\mathrm{i}\ \mathrm{Cos}\left[\frac{3\,\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\,\pi}{14}\right]\right)^{2}+$$

$$\mathrm{i}\ \Bigg(-2030\ \mathrm{i}+546\ \sqrt{91}-\frac{7\ (-\mathrm{i}+\sqrt{3})\ (35\,789\ \mathrm{i}+73\,629\ \sqrt{91})}{\left(6\,212\,842\,012\ \mathrm{i}-\frac{201\,901\,707\ \sqrt{91}}{2}+\frac{3}{2}\ \sqrt{39\ (-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\ \mathrm{i}\ \sqrt{91})}\right)^{1/3}}+$$

$$2^{2/3}\ (1-\mathrm{i}\ \sqrt{3})\ \Bigg(12\,425\,684\,024\ \mathrm{i}-201\,901\,707\ \sqrt{91}+$$

$$3\ \sqrt{39\ (-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\ \mathrm{i}\ \sqrt{91})}\Bigg)^{1/3}\Bigg)\ \left(\mathrm{i}\ \mathrm{Cos}\left[\frac{3\,\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\,\pi}{14}\right]\right)^{3}-$$

$$i\left(2030\, i+546\sqrt{91}+\frac{7\left(i+\sqrt{3}\right)\left(-35789\, i+73629\sqrt{91}\right)}{\left(6212842012\, i+\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139-3314618482457212\, i\sqrt{91}\right)}\right)^{1/3}}-\right.$$

$$2^{2/3}\left(1+i\sqrt{3}\right)\left(12425684024\, i+201901707\sqrt{91}+\right.$$

$$\left.\left.3\sqrt{39\left(-455168513200298139-3314618482457212\, i\sqrt{91}\right)}\right)^{1/3}\right)\left(i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^{4}+$$

$$i\left(-2030\, i+546\sqrt{91}+\frac{7\left(i+\sqrt{3}\right)\left(35789\, i+73629\sqrt{91}\right)}{\left(6212842012\, i-\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139+3314618482457212\, i\sqrt{91}\right)}\right)^{1/3}}+\right.$$

$$\left.2^{2/3}\left(1+i\sqrt{3}\right)\left(12425684024\, i-201901707\sqrt{91}+3\sqrt{39\left(-455168513200298139+3314618482457212\, i\sqrt{91}\right)}\right)^{1/3}\right)$$

$$\left(i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^{5}+2\, i\left(7\left(-145\, i+39\sqrt{91}\right)+\right.$$

$$\frac{7\left(35789-73629\, i\sqrt{91}\right)}{\left(6212842012\, i-\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139+3314618482457212\, i\sqrt{91}\right)}\right)^{1/3}}-$$

$$\left.2^{2/3}\left(12425684024\, i-201901707\sqrt{91}+3\sqrt{39\left(-455168513200298139+3314618482457212\, i\sqrt{91}\right)}\right)^{1/3}\right)$$

$$\left.\left.\left(i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^{6}\right)\right)^{1/7}+\left(\frac{7}{2}\right)^{2/7}\left(\frac{1}{3}\left(-12180-\right.\right.$$

$$i\left(2030\, i+546\sqrt{91}+\frac{7\left(i+\sqrt{3}\right)\left(-35789\, i+73629\sqrt{91}\right)}{\left(6212842012\, i+\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139-3314618482457212\, i\sqrt{91}\right)}\right)^{1/3}}-\right.$$

$$\left.2^{2/3}\left(1+i\sqrt{3}\right)\left(12425684024\, i+201901707\sqrt{91}+3\sqrt{39\left(-455168513200298139-3314618482457212\, i\sqrt{91}\right)}\right)^{1/3}\right)\left(i\right.$$

$$\left.\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)-$$

$$i\left(2030\, i+546\sqrt{91}+\frac{7\left(1+i\sqrt{3}\right)\left(35789+73629\, i\sqrt{91}\right)}{\left(6212842012\, i+\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139-3314618482457212\, i\sqrt{91}\right)}\right)^{1/3}}+\right.$$

$$i\,2^{2/3}\left(i+\sqrt{3}\right)\left(12425684024\, i+201901707\sqrt{91}+\right.$$

$$\left.\left.3\sqrt{39\left(-455168513200298139-3314618482457212\, i\sqrt{91}\right)}\right)^{1/3}\right)\left(i\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^{2}+$$

$$i\left(-2030\, i+546\sqrt{91}+\frac{7\left(i+\sqrt{3}\right)\left(35789\, i+73629\sqrt{91}\right)}{\left(6212842012\, i-\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139+3314618482457212\, i\sqrt{91}\right)}\right)^{1/3}}+\right.$$

$$\left.2^{2/3}\left(1+i\sqrt{3}\right)\left(12425684024\, i-201901707\sqrt{91}+3\sqrt{39\left(-455168513200298139+3314618482457212\, i\sqrt{91}\right)}\right)^{1/3}\right)$$

$$\left(\mathrm{i}\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^3-2\,\mathrm{i}\left(7\left(145\,\mathrm{i}+39\sqrt{91}\right)-\frac{7\left(35789+73629\,\mathrm{i}\sqrt{91}\right)}{\left(6212842012\,\mathrm{i}+\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139-3314618482457212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}}+2^{2/3}\left(12425684024\,\mathrm{i}+201901707\sqrt{91}+3\sqrt{39\left(-455168513200298139-3314618482457212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}\right)$$

$$\left(\mathrm{i}\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^4+2\,\mathrm{i}\left(7\left(-145\,\mathrm{i}+39\sqrt{91}\right)+\frac{7\left(35789-73629\,\mathrm{i}\sqrt{91}\right)}{\left(6212842012\,\mathrm{i}-\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139+3314618482457212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}}-2^{2/3}\left(12425684024\,\mathrm{i}-201901707\sqrt{91}+3\sqrt{39\left(-455168513200298139+3314618482457212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}\right)\left(\mathrm{i}\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^5+$$

$$\mathrm{i}\left(-2030\,\mathrm{i}+546\sqrt{91}-\frac{7\left(-\mathrm{i}+\sqrt{3}\right)\left(35789\,\mathrm{i}+73629\sqrt{91}\right)}{\left(6212842012\,\mathrm{i}-\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139+3314618482457212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}}+2^{2/3}\left(1-\mathrm{i}\sqrt{3}\right)\left(12425684024\,\mathrm{i}-201901707\sqrt{91}+3\sqrt{39\left(-455168513200298139+3314618482457212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}\right)$$

$$\left.\left.\left(\mathrm{i}\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^6\right)\right)^{1/7}-\frac{1}{2^{2/7}}\times 7^{2/7}\left(\frac{1}{3}\left(12180+\right.\right.$$

$$2\,\mathrm{i}\left(7\left(145\,\mathrm{i}+39\sqrt{91}\right)-\frac{7\left(35789+73629\,\mathrm{i}\sqrt{91}\right)}{\left(6212842012\,\mathrm{i}+\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139-3314618482457212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}}+2^{2/3}\left(12425684024\,\mathrm{i}+201901707\sqrt{91}+3\sqrt{39\left(-455168513200298139-3314618482457212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}\right)\left(\mathrm{i}\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)+$$

$$\mathrm{i}\left(2030\,\mathrm{i}+546\sqrt{91}+\frac{7\left(\mathrm{i}+\sqrt{3}\right)\left(-35789\,\mathrm{i}+73629\sqrt{91}\right)}{\left(6212842012\,\mathrm{i}+\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139-3314618482457212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}}-2^{2/3}\left(1+\mathrm{i}\sqrt{3}\right)\left(12425684024\,\mathrm{i}+201901707\sqrt{91}+3\sqrt{39\left(-455168513200298139-3314618482457212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}\right)$$

$$\left(\mathrm{i}\,\mathrm{Cos}\left[\frac{3\pi}{14}\right]+\mathrm{Sin}\left[\frac{3\pi}{14}\right]\right)^2-2\,\mathrm{i}\left(7\left(-145\,\mathrm{i}+39\sqrt{91}\right)+\frac{7\left(35789-73629\,\mathrm{i}\sqrt{91}\right)}{\left(6212842012\,\mathrm{i}-\frac{201901707\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455168513200298139+3314618482457212\,\mathrm{i}\sqrt{91}\right)}\right)^{1/3}}-\right.$$

$$2^{2/3}\Big(12\,425\,684\,024\,\mathrm{i}-201\,901\,707\,\sqrt{91}+3\,\sqrt{39\,\big(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,\mathrm{i}\,\sqrt{91}\big)}\Big)^{1/3}\Bigg)\Big(\mathrm{i}$$

$$\mathrm{Cos}\Big[\frac{3\,\pi}{14}\Big]+\mathrm{Sin}\Big[\frac{3\,\pi}{14}\Big]\Big)^{3}+$$

$$\mathrm{i}\Bigg(2030\,\mathrm{i}+546\,\sqrt{91}+\frac{7\,\big(1+\mathrm{i}\,\sqrt{3}\big)\,\big(35\,789+73\,629\,\mathrm{i}\,\sqrt{91}\big)}{\Big(6\,212\,842\,012\,\mathrm{i}+\frac{201\,901\,707\,\sqrt{91}}{2}+\frac{3}{2}\,\sqrt{39\,\big(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,\mathrm{i}\,\sqrt{91}\big)}\Big)^{1/3}}+$$

$$\mathrm{i}\,2^{2/3}\,\big(\mathrm{i}+\sqrt{3}\big)\,\Big(12\,425\,684\,024\,\mathrm{i}+201\,901\,707\,\sqrt{91}+$$

$$3\,\sqrt{39\,\big(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,\mathrm{i}\,\sqrt{91}\big)}\Big)^{1/3}\Bigg)\Big(\mathrm{i}\,\mathrm{Cos}\Big[\frac{3\,\pi}{14}\Big]+\mathrm{Sin}\Big[\frac{3\,\pi}{14}\Big]\Big)^{4}-$$

$$\mathrm{i}\Bigg(-2030\,\mathrm{i}+546\,\sqrt{91}-\frac{7\,\big(-\mathrm{i}+\sqrt{3}\big)\,\big(35\,789\,\mathrm{i}+73\,629\,\sqrt{91}\big)}{\Big(6\,212\,842\,012\,\mathrm{i}-\frac{201\,901\,707\,\sqrt{91}}{2}+\frac{3}{2}\,\sqrt{39\,\big(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,\mathrm{i}\,\sqrt{91}\big)}\Big)^{1/3}}+$$

$$2^{2/3}\,\big(1-\mathrm{i}\,\sqrt{3}\big)\,\Big(12\,425\,684\,024\,\mathrm{i}-201\,901\,707\,\sqrt{91}+$$

$$3\,\sqrt{39\,\big(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,\mathrm{i}\,\sqrt{91}\big)}\Big)^{1/3}\Bigg)\Big(\mathrm{i}\,\mathrm{Cos}\Big[\frac{3\,\pi}{14}\Big]+\mathrm{Sin}\Big[\frac{3\,\pi}{14}\Big]\Big)^{5}-$$

$$\mathrm{i}\Bigg(-2030\,\mathrm{i}+546\,\sqrt{91}+\frac{7\,\big(\mathrm{i}+\sqrt{3}\big)\,\big(35\,789\,\mathrm{i}+73\,629\,\sqrt{91}\big)}{\Big(6\,212\,842\,012\,\mathrm{i}-\frac{201\,901\,707\,\sqrt{91}}{2}+\frac{3}{2}\,\sqrt{39\,\big(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,\mathrm{i}\,\sqrt{91}\big)}\Big)^{1/3}}+$$

$$2^{2/3}\,\big(1+\mathrm{i}\,\sqrt{3}\big)\,\Big(12\,425\,684\,024\,\mathrm{i}-201\,901\,707\,\sqrt{91}+$$

$$3\,\sqrt{39\,\big(-455\,168\,513\,200\,298\,139+3\,314\,618\,482\,457\,212\,\mathrm{i}\,\sqrt{91}\big)}\Big)^{1/3}\Bigg)\Big(\mathrm{i}\,\mathrm{Cos}\Big[\frac{3\,\pi}{14}\Big]+\mathrm{Sin}\Big[\frac{3\,\pi}{14}\Big]\Big)^{6}\Bigg)\Bigg)^{1/7}\Bigg)$$

x1単独とは信じられないほど長い式です．最後に，お約束の近似値比較です．上が 累乗根表現の， 下が NSolve で求めた解です．ご覧のように一致しています．

In[∘]:=
```
N[{x1, x2, x3, x4, x5, x6, x7}] // Sort // Chop
```

Out[∘]=
```
{-1.3888, -1.0395 - 0.82652 i, -1.0395 + 0.82652 i, 0.736558 + 0.408262 i,
 0.736558 - 0.408262 i, 0.997338 + 0.855759 i, 0.997338 - 0.855759 i}
```

In[∘]:=
```
x /. NSolve[f[x] == 0, x] // Sort
```

Out[∘]=
```
{-1.3888, -1.0395 - 0.82652 i, -1.0395 + 0.82652 i, 0.736558 - 0.408262 i,
 0.736558 + 0.408262 i, 0.997338 - 0.855759 i, 0.997338 + 0.855759 i}
```

§4. 補足

F42_Solutions.nb で調べた「R1, R2, ⋯, R6 を直接扱う方法」と，ここで調べた「l_0, l_1, l_2, ⋯, l_6 (ただしR1=l_0+$l_1\zeta$+$l_2\ \zeta^2$ + ⋯+$l_6\ \zeta^6$) を使う方法」を比較します．まとめると次の3つとなります．

- (*a*) {R1, R6} + {R2, R5} + {R3, R4} と分け, R1 + R6, R1R6, ⋯, R3 + R4, R3R4に関する3次方程式を立て，次に2次方程式を解いてR1～R6を求める方法
- (*b*) {R1, R2, R4} + {R3, R5, R6} と分け, R1 + R2 + R4, R1R2 + R2R4 + R4R1, R1R2R4, ⋯, R3R5R6 に関する2次方程式を立て，次に3次方程式を解いてR1～R6を求める方法
- (*c*) {*l1, l2, l4*} + {*l3, l5, l6*} と分け，それぞれに対する3次方程式を立てて *l1*～*l6* を求める方法

(a), (b),(c)によるR1の値はそれぞれ次のようになります．((c)では ζ = Cos[2 Pi/7] + I Sin[2 Pi/7])

(a)の場合

$$\mathrm{R1a} = -\frac{2401}{6}\left(145+39\sqrt{13}\right) + \left(1372\times 7^{2/3}\left(116\,506+32\,289\sqrt{13}\right)\right)\Big/\left(3\left(\frac{1}{2}\left(-43\,982\,153\,605-12\,198\,286\,674\sqrt{13}+3\,\mathrm{i}\sqrt{3\left(554\,388\,682\,885\,501\,321+153\,759\,755\,291\,383\,332\sqrt{13}\right)}\right)\right)^{1/3}\right) + \frac{49}{3}\left(\frac{7}{2}\left(-43\,982\,153\,605-12\,198\,286\,674\sqrt{13}+3\,\mathrm{i}\sqrt{3\left(554\,388\,682\,885\,501\,321+153\,759\,755\,291\,383\,332\sqrt{13}\right)}\right)\right)^{1/3}$$

(b)の場合

$$\mathrm{R1b} = \left(2401\left(1\,670\,116+7^{2/3}\left(-1-\mathrm{i}\sqrt{3}\right)\left(13\left(914\,544\,806\,249\,111+136\,483\,676\,034\,\mathrm{i}\sqrt{3}\right)\right)^{1/3}+\mathrm{i}\,7^{2/3}\left(13\left(914\,544\,806\,249\,111-136\,483\,676\,034\,\mathrm{i}\sqrt{3}\right)\right)^{1/3}\left(\mathrm{i}+\sqrt{3}\right)\right)\right)\Big/\left(3\left(\frac{49}{3}\left(-7105+\frac{521\,143\times 91^{2/3}}{\left(-3\,581\,010\,217-136\,936\,446\,\mathrm{i}\sqrt{3}\right)^{1/3}}+\left(91\left(-3\,581\,010\,217-136\,936\,446\,\mathrm{i}\sqrt{3}\right)\right)^{1/3}\right)+\sqrt{\left(\frac{2401}{9}\left(-7105+\frac{521\,143\times 91^{2/3}}{\left(-3\,581\,010\,217-136\,936\,446\,\mathrm{i}\sqrt{3}\right)^{1/3}}+\left(91\left(-3\,581\,010\,217-136\,936\,446\,\mathrm{i}\sqrt{3}\right)\right)^{1/3}\right)^2-\frac{4802}{3}\left(1\,670\,116+7^{2/3}\left(-1-\mathrm{i}\sqrt{3}\right)\left(13\left(914\,544\,806\,249\,111+136\,483\,676\,034\,\mathrm{i}\sqrt{3}\right)\right)^{1/3}+\mathrm{i}\,7^{2/3}\left(13\left(914\,544\,806\,249\,111-136\,483\,676\,034\,\mathrm{i}\sqrt{3}\right)\right)^{1/3}\left(\mathrm{i}+\sqrt{3}\right)\right)\right)}\right)\right)$$

(c)の場合

$$
\begin{aligned}
\mathrm{R1c} = \frac{49}{12} \Bigg(& -12\,180 + \mathrm{i} \Bigg(-2030\,\mathrm{i} + 546\sqrt{91} - \big(7(-\mathrm{i}+\sqrt{3})(35\,789\,\mathrm{i} + 73\,629\sqrt{91})\big) \Big/ \Bigg(6\,212\,842\,012\,\mathrm{i} - \\
& \frac{201\,901\,707\sqrt{91}}{2} + \frac{3}{2}\sqrt{39\,(-455\,168\,513\,200\,298\,139 + 3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91})}\Bigg)^{1/3} + \\
& 2^{2/3}(1 - \mathrm{i}\sqrt{3}) \Bigg(12\,425\,684\,024\,\mathrm{i} - 201\,901\,707\sqrt{91} + \\
& 3\sqrt{39\,(-455\,168\,513\,200\,298\,139 + 3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91})}\Bigg)^{1/3}\Bigg)\zeta + \\
& 2\,\mathrm{i} \Bigg(7(-145\,\mathrm{i} + 39\sqrt{91}) + \big(7(35\,789 - 73\,629\,\mathrm{i}\sqrt{91})\big) \Big/ \Bigg(6\,212\,842\,012\,\mathrm{i} - \frac{201\,901\,707\sqrt{91}}{2} + \frac{3}{2} \\
& \sqrt{39\,(-455\,168\,513\,200\,298\,139 + 3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91})}\Bigg)^{1/3} - 2^{2/3}\Bigg(12\,425\,684\,024\,\mathrm{i} - \\
& 201\,901\,707\sqrt{91} + 3\sqrt{39\,(-455\,168\,513\,200\,298\,139 + 3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91})}\Bigg)^{1/3}\Bigg)\zeta^2 - \\
& 2\,\mathrm{i} \Bigg(7(145\,\mathrm{i} + 39\sqrt{91}) - \big(7(35\,789 + 73\,629\,\mathrm{i}\sqrt{91})\big) \Big/ \Bigg(6\,212\,842\,012\,\mathrm{i} + \frac{201\,901\,707\sqrt{91}}{2} + \frac{3}{2} \\
& \sqrt{39\,(-455\,168\,513\,200\,298\,139 - 3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91})}\Bigg)^{1/3} + 2^{2/3}\Bigg(12\,425\,684\,024\,\mathrm{i} + \\
& 201\,901\,707\sqrt{91} + 3\sqrt{39\,(-455\,168\,513\,200\,298\,139 - 3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91})}\Bigg)^{1/3}\Bigg)\zeta^3 + \\
& \mathrm{i} \Bigg(-2030\,\mathrm{i} + 546\sqrt{91} + \big(7(\mathrm{i}+\sqrt{3})(35\,789\,\mathrm{i} + 73\,629\sqrt{91})\big) \Big/ \Bigg(6\,212\,842\,012\,\mathrm{i} - \\
& \frac{201\,901\,707\sqrt{91}}{2} + \frac{3}{2}\sqrt{39\,(-455\,168\,513\,200\,298\,139 + 3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91})}\Bigg)^{1/3} + \\
& 2^{2/3}(1 + \mathrm{i}\sqrt{3}) \Bigg(12\,425\,684\,024\,\mathrm{i} - 201\,901\,707\sqrt{91} + \\
& 3\sqrt{39\,(-455\,168\,513\,200\,298\,139 + 3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91})}\Bigg)^{1/3}\Bigg)\zeta^4 - \\
& \mathrm{i} \Bigg(2030\,\mathrm{i} + 546\sqrt{91} + \big(7(1 + \mathrm{i}\sqrt{3})(35\,789 + 73\,629\,\mathrm{i}\sqrt{91})\big) \Big/ \Bigg(6\,212\,842\,012\,\mathrm{i} + \\
& \frac{201\,901\,707\sqrt{91}}{2} + \frac{3}{2}\sqrt{39\,(-455\,168\,513\,200\,298\,139 - 3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91})}\Bigg)^{1/3} + \\
& \mathrm{i}\,2^{2/3}(\mathrm{i} + \sqrt{3}) \Bigg(12\,425\,684\,024\,\mathrm{i} + 201\,901\,707\sqrt{91} + \\
& 3\sqrt{39\,(-455\,168\,513\,200\,298\,139 - 3\,314\,618\,482\,457\,212\,\mathrm{i}\sqrt{91})}\Bigg)^{1/3}\Bigg)\zeta^5 - \\
& \mathrm{i} \Bigg(2030\,\mathrm{i} + 546\sqrt{91} + \big(7(\mathrm{i}+\sqrt{3})(-35\,789\,\mathrm{i} + 73\,629\sqrt{91})\big) \Big/ \Bigg(6\,212\,842\,012\,\mathrm{i} +
\end{aligned}
$$

$$\frac{201\,901\,707\,\sqrt{91}}{2}+\frac{3}{2}\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,\mathbb{i}\,\sqrt{91}\right)}\Bigg)^{1/3}-$$
$$2^{2/3}\left(1+\mathbb{i}\,\sqrt{3}\right)\Bigg(12\,425\,684\,024\,\mathbb{i}+201\,901\,707\,\sqrt{91}+$$
$$3\,\sqrt{39\left(-455\,168\,513\,200\,298\,139-3\,314\,618\,482\,457\,212\,\mathbb{i}\,\sqrt{91}\right)}\Bigg)^{1/3}\Bigg)\,\zeta^6\Bigg)$$

l_1, l_2, … を扱う方法は, 5 次方程式の時は非常にうまく行き,
7 次方程式でもかなり簡単に解くことができました.

しかし7次の時は $R_k = a_k + b_k\,\zeta + c_k\,\zeta^2 + d_k\,\zeta^3 + e_k\,\zeta^4 + f_k\,\zeta^5 + g_k\,\zeta^6$ の形となるので

$$x_k = \left(a_1 + b_1\,\zeta + c_1\,\zeta^2 + d_1\,\zeta^3 + e_1\,\zeta^4 + f_1\,\zeta^5 + g_1\,\zeta^6\right)^{1/7} +$$
$$\left(a_2 + b_2\,\zeta + c_2\,\zeta^2 + d_2\,\zeta^3 + e_2\,\zeta^4 + f_2\,\zeta^5 + g_2\,\zeta^6\right)^{1/7}\zeta +$$
$$\cdots + \left(a_7 + b_7\,\zeta + c_7\,\zeta^2 + d_7\,\zeta^3 + e_7\,\zeta^4 + f_7\,\zeta^5 + g_7\,\zeta^6\right)^{1/7}\zeta^6$$

という非常に長い形になってしまいます.どうやら7次方程式の場合は「解法の簡潔さ」と
「答えの簡潔さ」のどちらを重視するかにより, 解法を変える必要があるようです.それよりも,
そもそも, これらの異なる表現が可能であることが, 驚きです.