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Set $P(X,0,0)$, $Q(0,Y,1)$, then because $PQ = 2$

$$PQ^2 = X^2 + Y^2 + 1 = 4 \qquad \therefore\ X^2 + Y^2 = 3 \qquad \cdots ①$$

Set a point $R(x,y,z)$ such that R devides a segment PQ in a ratio of $t : (1-t)$ $(0 < t < 1)$, then.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (1-t)\begin{pmatrix} X \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ Y \\ 1 \end{pmatrix} = \begin{pmatrix} (1-t)X \\ tY \\ t \end{pmatrix} \Longleftrightarrow \begin{cases} x = (1-t)X \\ y = tY \\ z = t \end{cases} \qquad \cdots ②$$

By means of ① and ②.

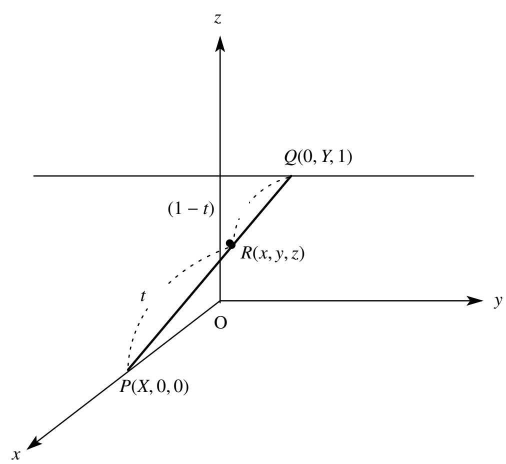
$$\left(\frac{x}{1-t}\right)^2 + \left(\frac{y}{t}\right)^2 = 3 \Longleftrightarrow \frac{x^2}{\{\sqrt{3}(1-t)\}^2} + \frac{y^2}{(\sqrt{3}t)^2} = 1 \qquad \cdots ③$$

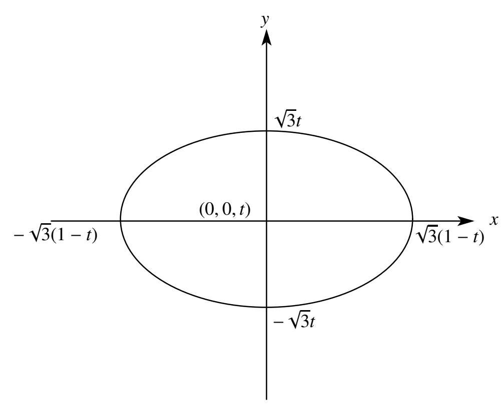
Therefore, the locus of the point R is a ellipse in the plane $z = t$.This is a crossing section of C by a plane $z = t$, thus the sectional area by $z = t$ $(0 < t < 1)$ is.

$$S(t) = \pi\sqrt{3}(1-t)\sqrt{3}t = 3\pi t(1-t) \qquad \cdots ④$$

Therfore, the volume is.

$$V = \int_0^1 S(t)\,dt = 3\pi\int_0^1 t(1-t)\,dt = \frac{\pi}{2} \qquad \cdots ans.$$





crossing section by $z = t$