17 Set three solutions of the equation as z_1, z_2 and z_3.Then,

$$\begin{cases} z_1 + z_2 + z_3 = 0 & \cdots ① \\ z_1 z_2 + z_2 z_3 + z_3 z_1 = -(3\sqrt{3} + 3i) & \cdots ② \\ z_1 z_2 z_3 = -\alpha & \cdots ③ \end{cases}$$

Because z_1, z_2 and z_3 are collinear, $z_1 + z_2 + z_3 = 0$ implies that z_1, z_2 and z_3 are in the same line which goes through O(0). Thus,we can set

$$z_1 = a\omega, z_2 = b\omega, z_3 = c\,\omega \ (|\omega| = 1,\ a, b, c \text{ are real numbers})$$

Then, by means of ①,② and ③.

$$\begin{cases} a + b + c = 0 & \cdots ①' \\ (ab + bc + ca)\omega^2 = -(3\sqrt{3} + 3i) & \cdots ②' \\ abc\,\omega^3 = -\alpha & \cdots ③' \end{cases}$$

By means of ①′

$$ab + bc + ca = ab - (a + b)^2 = -(a^2 + ab + b^2) \leq 0$$

If both sides are equal ,then $a = b = c = 0$ but it contradicts ②′,so $ab+bc+ca < 0$. Therefore ,by means of ②′.

$$\begin{cases} \arg\{(ab + bc + ca)\omega^2\} = 180^\circ + 2\arg\omega = \arg\{-(3\sqrt{3} + 3i)\} = 210^\circ \\ \left|(ab + bc + ca)\omega^2\right| = -(ab + bc + ca) = \left|-(3\sqrt{3} + 3i)\right| = 6 \end{cases}$$

$$\therefore \begin{cases} \arg\omega = 15^\circ \\ ab + bc + ca = -6 \end{cases} \cdots ④$$

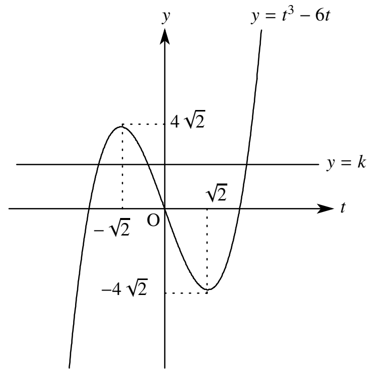
Therefore, if we set $abc = k$, then a, b and c are the three real number solutions of the equation below.

$$t^3 - 6t - k = 0 \Longleftrightarrow t^3 - 6t = k$$

Set $f(t) = t^3 - 6t$, then.

$$f'(t) = 3t^2 - 6 = 3(t - \sqrt{2})(t + \sqrt{2})$$

t	$\cdots$	$-\sqrt{2}$	$\cdots$	$\sqrt{2}$	$\cdots$
$f'(t)$	+	0	−	0	+
$f(t)$	↗	$4\sqrt{2}$	↘	$-4\sqrt{2}$	↗



Thus , if a, b, c are all real numbers ,the range of k is

$$-4\sqrt{2} \le k \le 4\sqrt{2} \qquad \cdots ⑤$$

By means of ③′, if $abc \ne 0 \Longleftrightarrow \alpha \ne 0$,then

$$\begin{cases} |\alpha| = |abc| \\ \arg\ \alpha = 180° + \arg\{abc\} + 3\arg\ \omega = 225° + \arg\{abc\} \end{cases}$$

By means of ④,⑤,if $\alpha \ne 0$,then.

$$0 < |\alpha| \le 4\sqrt{2},\ \arg\alpha = 225°\ or\ 45°$$

$\alpha = 0$ evidently satisfis the condition of the problem , so the existance regeon of α is the bold segment below.