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Set three solutions of $(*)$ as $x_1, x_2$ and $x_3$ ,then,

$$x_1 + x_2 + x_3 = 3\sqrt{3}$$

Therefore, if we set the conter of gravity of the equilateral triangle as $A(\alpha)$ then,

$$\alpha = \frac{x_1 + x_2 + x_3}{3} = \sqrt{3}$$

Here, if we translate the triangle so that $\alpha$ comes to O(0), three vertices of the triangle are

$$y_1 = x_1 - \sqrt{3},\ y_2 = x_2 - \sqrt{3},\ y_2 = x_2 - \sqrt{3}$$

Therefore,

$$x_1 = y_1 + \sqrt{3},\ x_2 = y_2 + \sqrt{3},\ x_3 = y_3 + \sqrt{3}$$

In accordance with that, $y_1 + \sqrt{3},\ y_2 + \sqrt{3},\ y_3 + \sqrt{3}$ are three solutions of $(*)$,then the equation which has solutions of $y_1,\ y_2$ and $y_3$ is,

$$(y + \sqrt{3})^3 - 3\sqrt{3}(y + \sqrt{3})^2 + a(y + \sqrt{3}) - 6\sqrt{3} = 0$$
$$\Longleftrightarrow y^3 + (a-9)y + (a-12)\sqrt{3} = 0 \quad \cdots ①$$

$\{y_1,\ y_2,\ y_3\}$ forms a equalateral triangle whose center of gravity is O(0).Therfore ① must be the equation such that $y^3 = k$ ($k$ is a real constant ).Then,

$$a - 9 = 0 \qquad \therefore\ a = 9 \qquad \cdots ans.$$

Substitute it to ①.

$$y^3 = 3\sqrt{3} \Longleftrightarrow y = \sqrt{3},\ \sqrt{3}\omega,\ \sqrt{3}\omega^2$$

Therefore, the solutions of $(*)$ are.

$$2\sqrt{3},\ \frac{\sqrt{3} \pm 3i}{2} \qquad \cdots ans.$$