$\boxed{2}$ Take the origin O at the center of sector D, take x axis, y axis and z axis to the direction of the south,east and right above respectively. Then the direction vector of the sunlight is $(-1, -1, -\sqrt{2})$. Thus **if there is no wall** the shadow of D is the translation of D in the direction of $(-1, -1, -\sqrt{2})$. Therfore, a cross section of E by $z = -\sqrt{2}t (t \geq 0)$ is a quarter sector with the center of $(-t, -t, -\sqrt{2}t)$ $(x \geq 0, y \geq 0)$.
Take an angle $\theta (0 \leq \theta \leq \frac{\pi}{4})$ as below, and let S be a cross sectional area of E by $z = -\sqrt{2}t (t \geq 0)$, then :

$$t = \sin\theta$$

$$z = -\sqrt{2}t = -\sqrt{2}\sin\theta$$

$$S = \frac{1}{2}\cdot 1^2 \cdot 2\left(\frac{\pi}{4} - \theta\right) - \frac{1}{2}t(\cos\theta - t)\times 2 \qquad \text{sector}-\text{triangle}$$

$$= \left(\frac{\pi}{4} - \theta\right) - \sin\theta(\cos\theta - \sin\theta)$$

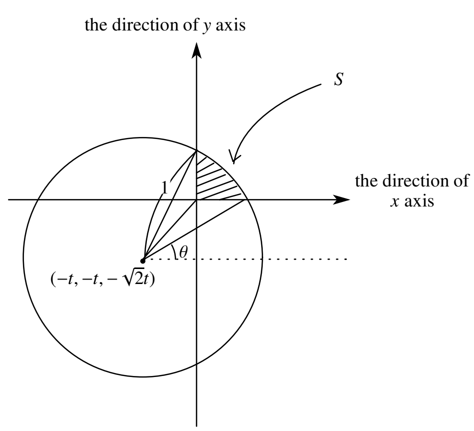
$$\therefore V = \int_{-1}^{0}\left(\frac{\pi}{4} - \theta + \sin^2\theta - \sin\theta\cos\theta\right)dz$$

$$= \sqrt{2}\int_{0}^{\frac{\pi}{4}}\left(\frac{\pi}{4} - \theta + \sin^2\theta - \sin\theta\cos\theta\right)\cos\theta d\theta \qquad \begin{array}{c|cc} z & -1 & \to 0 \\ \hline \theta & \frac{\pi}{4} & \to 0 \end{array}$$

$$= \frac{\sqrt{2}\pi}{4}\int_{0}^{\frac{\pi}{4}}\cos\theta d\theta - \sqrt{2}\int_{0}^{\frac{\pi}{4}}\theta\cos\theta d\theta + \sqrt{2}\int_{0}^{\frac{\pi}{4}}\sin^2\theta\cos\theta d\theta - \sqrt{2}\int_{0}^{\frac{\pi}{4}}\sin\theta\cos^2\theta d\theta$$

$$= \frac{\sqrt{2}\pi}{4}\Big[\sin\theta\Big]_{0}^{\frac{\pi}{4}} - \sqrt{2}\Big[\theta\sin\theta + \cos\theta\Big]_{0}^{\frac{\pi}{4}} + \frac{\sqrt{2}}{3}\Big[\sin^3\theta + \cos^3\theta\Big]_{0}^{\frac{\pi}{4}}$$

$$= \frac{2\sqrt{2}}{3}\pi - \frac{2}{3} \qquad \cdots ans.$$



Comment

I came up with this problem when I saw a small shelf in a bathroom.