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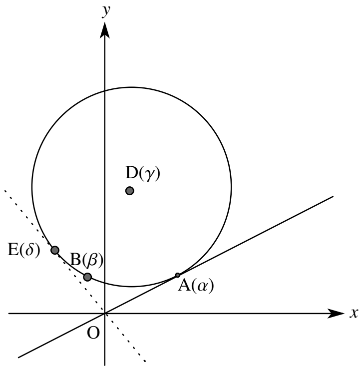


(1) $\dfrac{\beta-\alpha}{0-\alpha} = \dfrac{(-1-\sqrt{6})+i(\sqrt{2}-\sqrt{3})}{-\sqrt{3}(\sqrt{2}+i)}$

$= \dfrac{(1+\sqrt{6})+i(\sqrt{3}-\sqrt{2})}{\sqrt{3}(\sqrt{2}+i)} \cdot \dfrac{\sqrt{2}-i}{\sqrt{2}-i}$

$= \dfrac{2}{\sqrt{3}}\left(\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i\right)$

$= \dfrac{2}{\sqrt{3}}\{\cos(-30^\circ)+i\sin(-30^\circ)\}$

Therfore,

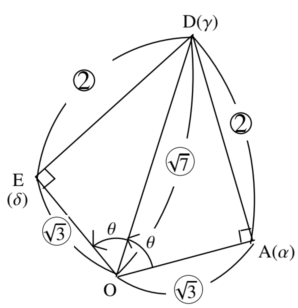
$$\overrightarrow{AO} \xrightarrow[(-30^\circ)\text{rotate}]{\text{enlarge by the ratio of } \frac{2}{\sqrt{3}}} \overrightarrow{AB} \quad \therefore \quad \angle OAB = 30^\circ \qquad \cdots ans.$$

(2) Because $\overrightarrow{AD} \perp \overrightarrow{AO}$, $\angle BAD = 60^\circ$. Then,$\triangle ABD$ is equilateral.

$$\overrightarrow{AB} \xrightarrow[(-60^\circ)\text{rotate}]{} \overrightarrow{AD}.$$

Then,

$$\gamma-\alpha = (\beta-\alpha)\{\cos(-60^\circ)+i\sin(-60^\circ)\}$$
$$= \left\{(-1-\sqrt{6})+i(\sqrt{2}-\sqrt{3})\right\}\left(\frac{1-\sqrt{3}\,i}{2}\right) = -2+2\sqrt{2}\,i$$
$$\therefore \quad \gamma = (-2+2\sqrt{2}\,i)+\alpha = (\sqrt{6}-2)+i(2\sqrt{2}+\sqrt{3}) \qquad \cdots ans.$$



(3) In $\triangle OAD$, by maeans of Pythagoras'thorem,

$$\text{OA} : \text{AD} : \text{DO} = \sqrt{3} : 2 : \sqrt{7}$$

Then, if we set $\angle AOD = \theta$,

$$\begin{cases} \sin\theta = \dfrac{2}{\sqrt{7}} \\ \cos\theta = \dfrac{\sqrt{3}}{\sqrt{7}} \end{cases}$$

Considering $\triangle OAD \equiv \triangle OED$

$$\overrightarrow{OD} \xrightarrow[\theta\text{ rotate}]{\text{enlarge at the ratio of } \frac{\sqrt{3}}{\sqrt{7}}} \overrightarrow{OE}$$

$$\therefore \quad \delta = \gamma \times \frac{\sqrt{3}}{\sqrt{7}}(\cos\theta + i\sin\theta) = \left\{(\sqrt{6}-2)+i(2\sqrt{2}+\sqrt{3})\right\} \times \frac{\sqrt{3}}{\sqrt{7}}\left(\frac{\sqrt{3}}{\sqrt{7}}+i\frac{2}{\sqrt{7}}\right)$$
$$= \frac{(-\sqrt{6}-12)+i(12\sqrt{2}-\sqrt{3})}{7} \qquad \cdots ans.$$